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Communication in Parallel Systems

Fall 2000

Midterm Exam

Problem 1: (2 points)
Define the function of a switching algorithm and a routing algorithm.

Problem 2: (3 points)
Define the expansion and the routing number of a graph. How do these relate with each other?

Problem 3: (3 points)
Define the congestion and the dilation of a path collection and prove that the time to send packets along a path collection of congestion C and dilation D can be at least $C + D$ (not just at least $C + D - 1$!) in the worst case. It suffices to give an example.

Problem 4: (3 points)
What is oblivious routing? Describe Valiant's trick and explain why it can help to improve oblivious routing.

Problem 5: (3 points)
Show that for any network of routing number R there is a system of paths, one path for each pair of nodes, so that when selecting paths out of it with the help of Valiant's trick, for every permutation we obtain a path collection with dilation and expected congestion at most $O(R)$. Hint: use the definition of R to construct a path system out of path collections for permutations.

Problem 6: (2 points)
In which cases do we know that packets can be sent online along a path collection with congestion C and dilation D in $O(C + D + \log n)$ steps w.h.p.?

Problem 7: (4 points)
For any $d \in \mathbb{N}$, the *d-dimensional butterfly* $BF(d)$ is an undirected graph with node set $V = [d] \times [2]^d$ and edge set $E = \{\{(i, \alpha), ((i + 1) \bmod d, \beta)\} : i \in [d] \text{ and } \alpha = \beta \text{ or } \alpha \text{ and } \beta \text{ differ only in the } i\text{th bit position}\}$ (we define here $[k] = \{0, \ldots, k - 1\}$).

(a) Suggest an oblivious path selection strategy for this class of networks.

(b) Describe an online switching strategy with runtime $O(\text{congestion}+\text{dilation}+\log n)$ w.h.p. for sending the packets along the paths chosen in (a), where n is the size of the network.

(c) (2 bonus points) Bound the runtime of the switching strategy if each node wants to send one packet to a random destination.

Good luck!