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Communication in Parallel Systems

Fall 2000

Final Exam

Problem 1: Sorting networks (4 points)

1. What is the 0-1-principle? (1 point)
2. Design a sorting network that is not directly related to Batcher's sorting network. Explain, why it sorts. (3 points)

Problem 2: Network simulations (8 points)

1. What is an (n, k)-universal network? (1 point)
2. Show that the hypercube of size n is an $(n \log n, \log n)$-universal network. (4 points)
   Hint: bound the congestion and dilation of routing the requests. Conclude from this the slowdown of the simulation.
3. What is the lowest upper bound that can be achieved for k so that there is a constant degree network that is an (n, k)-simulator? Why? (3 points)

Problem 3: PRAM simulations (4 points)

1. What is a c-Collision DMM? (1 point)
2. Show that any read or write step of a CRCW PRAM can be simulated by an Arbitrary DMM in $O(\log n / \log \log n)$ steps with high probability. (3 points)
   Hint: we allow each memory module to have a communication window in which it can place a memory cell that can be read simultaneously by an arbitrary number of processors.

Problem 4: Hashing (4 points)

1. What is a k-univeral class of hash functions? (1 point)

2. Consider the following data placement strategy:

   We are given k random hash functions $h_i : U \to [0, 1)$. The storage devices are numbered in a consecutive way from 0 to $n-1$. Device d gets the interval $[f(d), f(d)+1/(2n))$, where $f(d) = \sum_{i \geq 0} d_i/2^{i+1}$ (d_i is bit i in the binary representation of d). A block b is placed at the device d with minimum i such that $h_i(b) \in [f(d), f(d) + 1/(2n))$. If no device fulfills this, b is stored in device 0.

   Show that none of the intervals assigned to the devices overlap. Furthermore show that the expected fraction of blocks placed at some device $d \geq 1$ is at most $1/n$. (3 points)

3. Prove that in the placement strategy above the expected fraction of blocks placed at device 0 is at most $1/n + 1/2^k$. (2 extra points)

Good luck!