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Communication in Parallel Systems

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Assignment 5

Problem 9 (8 points)**:**
Prove that the growing rank protocol described below is able to send packets along an arbitrary collection of n shortest paths with congestion C and dilation D in an arbitrary network in at most $O(C + D + \log n)$ time steps, with high probability.

Growing rank protocol:
At the beginning, every packet i chooses uniformly and independently at random an integer r_i out of some range $[K]$, where $K = \Theta(C)$ is sufficiently large. Let the *rank* of packet i that has already traversed ℓ edges be defined as $\text{rank}(i) = r_i + (K/D) \cdot \ell + \frac{i}{n+1}$. Then the growing rank protocol works as follows:

> For every time step and every edge with nonempty buffer, select the packet with minimum rank waiting at it and send it along that edge.

Hints for the analysis: Use the same delay sequence argument as for the random rank protocol. How large can the sum of the ℓ_i be at most? How does the runtime of the algorithm relate to the length of the delay sequence? Why does the protocol ensure that no packet can appear twice in an active delay sequence?