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Communication in Parallel Systems

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Assignment 3

Problem 5 (2 points)**:**
Prove Theorem 3.2. Use the following class of graphs for the proof.

Definition. The *b-ary DeBruijn graph of dimension d* $DB(b,d)$ is an undirected graph $G = (V,E)$ with node set $V = \{v \in [b]^d\}$ and edge set E that contains all edges $\{v,w\}$ with the property that $w \in \{(x, v_{d-1}, \ldots, v_1) : x \in [b]\}$, where $v = (v_{d-1}, \ldots, v_0)$.

Two examples of a DeBruijn graph can be found in Figure 1.

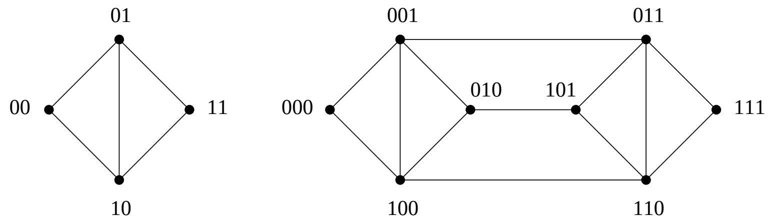

Figure 1: The structure of $DB(2,2)$ and $DB(2,3)$.

Problem 6 (2 points)**:**
Use the results in the lecture to show that there are graphs of routing number R and expansion α^{-1} with the property that $R = \Omega(\alpha^{-1} \log n)$.

Problem 7 (4 points)**:**
Prove that for every graph G of routing number R,

$$R = \Theta(\bar{R}), \text{ where } \bar{R} = \frac{1}{|S_n|} \sum_{\pi \in S_n} R(G, \pi)$$

is the average number of steps to route a permutation using a best possible strategy. Hint: use a strategy similar to Valiant's trick together with the Markov inequality to bound the best possible time required for an arbitrary permutation in terms of $\bar{R}$.

Definition (Markov Inequality). For every positive random variable X and every $k > 0$,

$$\Pr[X \geq k] \leq \frac{\mathrm{E}[X]}{k} .$$