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Communication in Parallel Systems

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Assignment 2

Problem 3 (4 points)**:**
Generalize the Borodin-Hopcroft lower bound to show that for every graph of size n and degree d and oblivious routing strategy, there is a partial permutation of size m (i.e., m source-destination pairs) that causes a congestion of $\Omega(\frac{m}{\sqrt{d \cdot n}})$.

Definition. A graph $G = (V, E)$ is called *vertex transitive* if for every pair of nodes $v, w \in V$ there is a permutation $\pi : V \to V$ mapping v to w such that for the graph $G' = (V, E')$ with $E' = \{\{x, y\} : \{\pi^{-1}(x), \pi^{-1}(y)\} \in E\}$ it holds that $G' = G$.

Vertex transitive graphs can be viewed as "looking the same" from any node. Examples of vertex transitive graphs are the cycle, torus, and hypercube.

Problem 4 (4 points)**:**
Suppose that we have an arbitrary vertex transitive graph G of constant degree. We want to use the following path selection strategy for G: for every source-destination pair (s, t), select a path $(v_0 = s, v_1, \ldots, v_{d-1}, v_d = t)$ from s to t by choosing uniformly at random at every node v_i a node v_{i+1} among all neighbors of v_i that are on a shortest path from v_i to t. Let D be the length of the longest path that can be created in this way. Obviously, D is equal to the diameter of G.
Suppose now that every node wants to send one packet to a random destination. Show (via symmetry arguments) that in this case the expected congestion at every node is the same, and prove an upper bound on this congestion, using the parameter D. Conclude from this an upper bound for the expected congestion at any fixed node when using Valiant's trick to route an arbitrary permutation.