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Communication in Parallel Systems

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Assignment 1

Definition. Let $m, d \in \mathbb{N}$. The (m,d)-*mesh* $M(m,d)$ is a graph with node set $V = [m]^d$ ($[m]$ represents here the set $\{0,\ldots,m-1\}$) and edge set

$$E = \left\{ \{(a_{d-1}\ldots a_0),(b_{d-1}\ldots b_0)\} \mid a_i, b_i \in [m],\ \sum_{i=0}^{d-1} |a_i - b_i| = 1 \right\} .$$

The (m,d)-*torus* $T(m,d)$ consists of an (m,d)-mesh and additionally an edge from $(a_{d-1}\ldots a_{i+1}0\, a_{i-1}\ldots a_0)$ to $(a_{d-1}\ldots a_{i+1}(m-1)a_{i-1}\ldots a_0)$ for every $i \in [d]$ and $a_j \in [m]$. $M(m,1)$ is also called *linear array*, $T(m,1)$ *cycle*, and $M(2,d) = T(2,d)$ *d-dimensional hypercube.*

Figure 1 presents a linear array, a torus, and a hypercube.

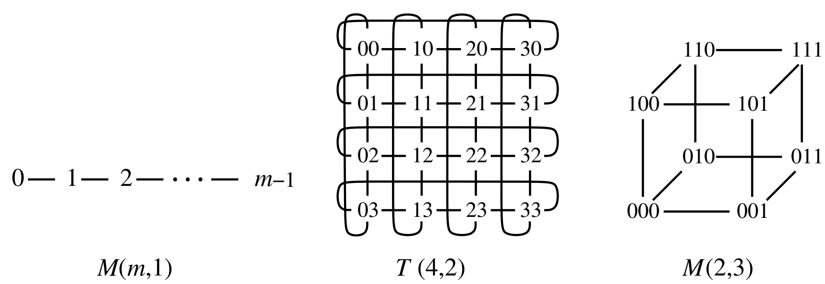

Figure 1: The structure of $M(m,1)$, $T(4,2)$, and $M(2,3)$.

Definition. A *permutation problem* is a routing problem in which packets are given whose source-destination pairs have the property that every node of the communication graph is exactly once the source and the destination of a packet.

Problem 1 (2 points)**:**
Show that any permutaton problem on an $(n,2)$-torus can be routed in at most $3n$ steps.

Problem 2 (4 points)**:**
Show that any permutation problem on a d-dimensional hypercube can be routed in $O(2^{d/2})$ steps.
Hint: for any source-destination pair $(s_{d-1},\ldots,s_0),(t_{d-1},\ldots,t_0) \in \{0,1\}^d$, use the path that first adjusts s_0 to t_0, then s_1 to t_1, and so on, until all bits have been set to the destination's values. Bound the congestion along a path carefully!