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Approximation Algorithms

Fall 2001

Assignment 7

Problem 21 (5 points)**:**
Let $G = (V, E)$ be a graph with vertex set $V = \{v_1, \ldots, v_n\}$. Recall that a *vertex cover* of G is a set $C \subseteq V$ with the property that for every edge $\{v_i, v_j\} \in E$, either v_i or v_j is in C. In the vertex cover problem (VC) the task is to find the smallest possible vertex cover.

(a) Explain why the following IP is an arithmetization of VC: (2 points)

Integer program for VC:

$$\begin{aligned} \text{minimize} \quad & \sum_{i=1}^{n} x_i \\ \text{subject to} \quad & x_i + x_j \geq 1 \qquad \text{for all } \{v_i, v_j\} \in E \\ & x_i \in \{0, 1\} \qquad \text{for all } v_i \in V \end{aligned}$$

(b) Show that a deterministic rounding of the optimal solution to the linear relaxation of this IP according to the rule "if $x_i \geq 1/2$, then add v_i to C" results in an approximation ratio of at most 2. Note that for this one has to show that the rule above results in a feasible solution. (3 points)

Problem 22 (2 points)**:**
Show that Corollary 6.3 is best possible (i.e. there exists a Boolean expression in CNF for which at most half of its clauses can be satisfied).

Problem 23 (3 points)**:**
Show that when using $\pi_i = \frac{1}{2}x_i^* + \frac{1}{4}$ for every i, then algorithm B has an expected approximation ratio of at most 4/3.