Dr. Christian Scheideler
Johns Hopkins University

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Approximation Algorithms

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Assignment 4

Problem 12 (3 points):
Show that NearestInsertion guarantees on every input $I = \langle K_n, c \rangle$ an approximation ratio of $2 - 2/n$.

Hint: Let the set of nodes in K_n be denoted by V. Prim's algorithm for computing a minimum spanning tree $T = (V_T, E_T)$ in K_n works as follows:

$V_T = \{v\}$, where v is an arbitrary node in V
$E_T = \emptyset$
while $|V_T| < n$ do
    select an $e = (u, v) \in V_T \times (\Gamma(V_T) \setminus V_T)$ with minimum $c(e)$
    $V_T = V_T \cup \{v\}$
    $E_T = E_T \cup \{e\}$

Show that for all j, $\text{cost}(v_j) \leq 2 \cdot c(e_j)$ for the edge e_j that is selected by Prim's algorithm as the jth edge of the MST T for I.

Problem 13 (2 points):
Show that in every tree the number of nodes of odd degree is even. (Hint: use induction.)

Problem 14 (2 points):
If one has a TSP that works with purely Euclidean distances, then one can quickly get into rounding problems due to the use of the square root. A distance may not have a finite representation any more. Show that when using instead the distance function

$$c(u_1, u_2) = \left\lceil \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \right\rceil$$

for any two points $u_1 = (x_1, y_1)$ and $u_2 = (x_2, y_2)$, then the triangle inequality is still fulfilled.

Problem 15 (3 points):
Show that Christofides' algorithm can produce an output for the family of graphs given in Figure 1 with even n that can be away by a factor of $\frac{3}{2} - \frac{1}{n}$ from an optimal round trip. All given edges are assumed to have a length of 1, and the length of all omitted edges is determined by the rounded Euclidean distance function in Problem 14.

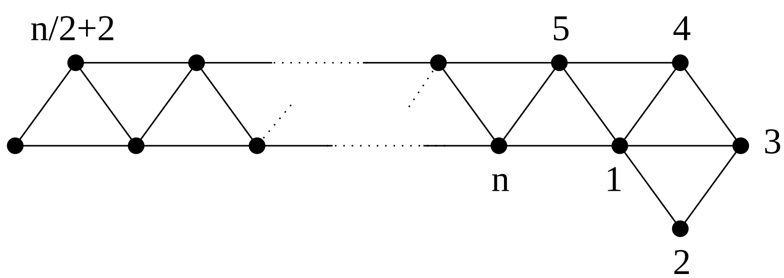

Figure 1: A bad graph G_{bad} for CH.