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Approximation Algorithms

Fall 2001

Assignment 3

Problem 8 (3 points)**:**
Show that for $\Delta \in \{2, 3\}$ the algorithm ColorEdges presented in the proof of Vizing's Theorem (see the updated version on the webpage!) has an absolute deviation of 1 (i.e. provide an infinite family of edge-Δ-colorable graphs for which the algorithm uses $\Delta + 1$ colors).

Problem 9 (2 points)**:**
Use the insights from the previous problem to construct an edge coloring algorithm that guarantees an approximation ratio of 4/3.

Problem 10 (3 points)**:**
Determine a maximum possible function $f(|I|)$ (where I is an instance) with the property that if one can approximate every Knapsack instance up to an absolute error of $f(|I|)$ then P = NP. Justify your selection.

Problem 11 (2 points)**:**
Let Π be a maximization problem and A be an approximation algorithm for Π. The *asymptotic approximation ratio* of A is defined as

$$\rho_A^\infty(n) = \min_{\mathbb{R}^{\mathbb{N}}}\{r(n) \mid \text{there is an } \ell \in \mathbb{N} \text{ so that for all } I \text{ with } \mathrm{OPT}(I) \geq \ell\text{: } \rho_A(I) \leq r(|I|)\}$$

Suppose that Π represents maxTSP and $\rho_A^\infty(n)$ is at most some polynomial in n. Show that in this case one can construct an algorithm for Π that guarantees an approximation ratio of $O(\rho_A^\infty(|I|))$ for *all* I.