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Modern Complexity Theory

Spring 2004

Midterm Exam

Problem 1: Completeness (7 points)

(a) Consider any C-complete language L for some complexity class C. Show that for every language $L' \in C$ with $L \leq_p L'$ it holds that also L' is C-complete. (2 points)

(b) Recall that $QSAT = \{\langle\psi\rangle \mid \psi$ is a valid quantified Boolean expression in conjunctive prenex normal form$\}$. Consider now the decision problem

$$GAME = \{\langle\psi\rangle \mid \psi \text{ is a valid quantified Boolean expression of the form } \exists x_1 \forall x_2 \exists x_3 \ldots Q_n x_n \phi \text{ for some } n \in \mathbb{N}, \text{ where } \phi \text{ is an expression in CNF with variables } x_1, \ldots, x_n\}$$

Show that $GAME$ is PSPACE-complete. (2 points)

(c) Show that PARITH restricted to variables with values in the range $\{0, \ldots, k\}$ for some constant $k \geq 1$ is PSPACE-complete. (3 points)

Problem 2: Undecidability (3 points)
Use the fact that the language

$$L_d = \{\langle M\rangle \mid M \text{ started with } \langle M\rangle \text{ does not accept}\}$$

is undecidable to show that also the language

$$E = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \Sigma^*\}$$

is undecidable.

Problem 3: Set Theory, Logic, and Arithmetic (6 points)

(a) Show that $|\mathbb{R}| = |\mathbb{R}^2|$.

(b) Show that for all quantified Boolean expressions ϕ_1 and ϕ_2 it holds that

$$\forall x\ (\phi_1 \wedge \phi_2) \equiv (\forall x\ \phi_1 \wedge \forall x\ \phi_2)\ .$$

(c) Give an *elementary* arithmetical expression for the statement "x is the greatest common divisor of a and b".

Good luck!