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Complexity Theory

Spring 2003

Midterm Exam

Problem 1: Complexity Classes (8 points)

1. Show that BPP $\subseteq$ PSPACE.
2. Show that NPSPACE $\subseteq$ EXP. (Hint: look at Lemma 3.6 and Savitch's Theorem.)
3. Consider any C-complete language L for some complexity class C. Show that for every language $L' \in C$ with $L \leq_p L'$ it holds that also L' is C-complete.
4. Show that the language SDE (solvable Diophantine equations) restricted to variables x in the range $\{0, \ldots, K\}$ for some fixed K is in NP.

Problem 2: Complexity of Logic (6 points)

1. Show that for all Boolean expressions ϕ_1, ϕ_2, and ϕ_3 it holds that
$$((\phi_1 \vee \phi_2) \wedge \phi_3) \equiv ((\phi_1 \wedge \phi_3) \vee (\phi_2 \wedge \phi_3)) .$$
2. Show that for all quantified Boolean expressions ϕ_1 and ϕ_2 it holds that
$$\exists x \ (\phi_1 \vee \phi_2) \equiv (\exists x \ \phi_1 \vee \exists x \ \phi_2) .$$
3. Transform $\forall x_1 \forall x_2 \exists x_3 \forall x_4 \ (x_1 \wedge (x_2 \vee \neg x_4)) \wedge (x_3 \vee x_4)$ into an equivalent arithmetical sentence with variables $x \in \{0, 1\}$. (A proof of equivalence is not needed.)

Problem 3: Undecidability (2 points)

Use the fact that the language
$$L = \{\langle M \rangle w \mid M \text{ started with } w \text{ accepts}\}$$
is undecidable to show that also the language
$$E = \{\langle M \rangle \mid M \text{ is a TM and } |L(M)| = 1\}$$
is undecidable.

Problem 4: Complexity of Arithmetics (4 points)

Give *elementary* arithmetical expressions for the following statements:

1. $x \in \mathbb{N}$ encodes a word $w \in \Sigma^*$ (in p-ary form) that contains the symbol c.
2. $x \in \mathbb{N}$ is a prime number.

Good luck!