Dr. Christian Scheideler
Johns Hopkins University

Baltimore, March 7 2005

Modern Complexity Theory

Spring 2005

Midterm Exam

Problem 1: Diverse Topics (8 points)

(a) Show that $|\mathbb{N}| = |\mathbb{N}^3|$. (2 points)

(b) Show that if L_1 and L_2 are recursively enumerable then also $L_1 \cap L_2$ is recursively enumerable. (2 points)

(c) Show that there is an infinite hierarchy for non-deterministic time. (2 points)

(d) Give an *elementary* arithmetical expression for the statement "x is the smallest common multiple of a and b". (2 points)

Problem 2: Complexity Classes (4 points)
Recall that C^O is the class of all decision problems that are in C if the Turing machines are allowed to use oracle O. Our aim is to show that $\mathrm{P}^{QSAT} = \mathrm{NP}^{QSAT}$. We do this in two steps.

(a) Show that $\mathrm{P}^{QSAT} = \mathrm{PSPACE}$. (Hint: remember to show "$\subseteq$" in both directions, and use the fact that QSAT is PSPACE-complete.) (2 points)

(b) Show that $\mathrm{NP}^{QSAT} = \mathrm{PSPACE}$. (Hint: extend the proof of NP $\subseteq$ PSPACE.) (2 points)

Problem 3: Undecidability (3 points)
Use the fact that the language $L_a = \{\langle M\rangle w \mid M \text{ started with } w \text{ accepts}\}$ is undecidable to show that also the language

$$E = \{\langle M_1\rangle\langle M_2\rangle \mid L(M_1) = L(M_2)\}$$

is undecidable. (Make sure your proof is complete. That is, just sketching f is not sufficient.)

Problem 4: Logic (5 points)

(a) Show that for all quantified Boolean expressions ϕ_1 and ϕ_2 it holds that (2 points)

$$\forall x\ (\phi_1 \wedge \phi_2) \equiv (\forall x\ \phi_1 \wedge \forall x\ \phi_2)\ .$$

(b) Transform the expression

$$(\exists x_1((x_1 \wedge x_3) \vee \neg\forall x_2(x_1 \wedge x_2))) \wedge (\forall x_3(x_3 \vee \exists x_4(x_1 \vee x_4)))$$

into prenex normal form using the equivalences in Section 4. (Hint: Be careful about the variables a quantifier applies to when moving it outside of the Boolean expression. Sometimes, substitutions have to be used to maintain equivalence.) (3 points)

Good luck!