

Modern Complexity Theory
Spring 2005
Midterm Exam

Problem 1: Diverse Topics (8 points)

- (a) Show that $|\mathbb{N}| = |\mathbb{N}^3|$. (2 points)
- (b) Show that if L_1 and L_2 are recursively enumerable then also $L_1 \cap L_2$ is recursively enumerable. (2 points)
- (c) Show that there is an infinite hierarchy for non-deterministic time. (2 points)
- (d) Give an *elementary* arithmetical expression for the statement “ x is the smallest common multiple of a and b ”. (2 points)

Problem 2: Complexity Classes (4 points)

Recall that C^O is the class of all decision problems that are in C if the Turing machines are allowed to use oracle O . Our aim is to show that $P^{QSAT} = NP^{QSAT}$. We do this in two steps.

- (a) Show that $P^{QSAT} = PSPACE$. (Hint: remember to show “ \subseteq ” in both directions, and use the fact that QSAT is PSPACE-complete.) (2 points)
- (b) Show that $NP^{QSAT} = PSPACE$. (Hint: extend the proof of $NP \subseteq PSPACE$.) (2 points)

Problem 3: Undecidability (3 points)

Use the fact that the language $L_a = \{\langle M \rangle w \mid M \text{ started with } w \text{ accepts}\}$ is undecidable to show that also the language

$$E = \{\langle M_1 \rangle \langle M_2 \rangle \mid L(M_1) = L(M_2)\}$$

is undecidable. (Make sure your proof is complete. That is, just sketching f is not sufficient.)

Problem 4: Logic (5 points)

- (a) Show that for all quantified Boolean expressions ϕ_1 and ϕ_2 it holds that (2 points)

$$\forall x (\phi_1 \wedge \phi_2) \equiv (\forall x \phi_1 \wedge \forall x \phi_2) .$$

- (b) Transform the expression

$$(\exists x_1((x_1 \wedge x_3) \vee \neg \forall x_2(x_1 \wedge x_2))) \wedge (\forall x_3(x_3 \vee \exists x_4(x_1 \vee x_4)))$$

into prenex normal form using the equivalences in Section 4. (Hint: Be careful about the variables a quantifier applies to when moving it outside of the Boolean expression. Sometimes, substitutions have to be used to maintain equivalence.) (3 points)

Good luck!