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Complexity Theory

Spring 2003

Final Exam

Problem 1: Interactive Proofs (6 points)

1. Explain why IP systems and PCP systems are different. (2 points)
2. Suppose that one of the following assertions were true:
   (a) For all classes $\mathcal{F}$ of functions, $\text{PCP}(\mathcal{F},\mathcal{F}) \subseteq \text{IP}(\mathcal{F})$.
   (b) For all classes $\mathcal{F}$ of functions, $\text{IP}(\mathcal{F}) \subseteq \text{PCP}(\mathcal{F},\mathcal{F})$.

   Show that then either P = NP or PSPACE = NEXPTIME. (4 points)

Problem 2: Approximability (6 points)

(a) What consequences does it have for the approximability of a problem P if there is an L-reduction from MAX3SAT to P? Justify your answer. (2 points)

(b) In the MAX2SAT problem we are given a Boolean expression in 2CNF, and the goal is to maximize the number of satisfied clauses. In the NAESAT problem we are given a Boolean expression in 3CNF, and the problem is find an assignment that maximizes the number of clauses that have at least one true and one false literal. Construct an L-reduction from MAX2SAT to NAESAT and try to bound the α and β in the L-reduction. (Hint: use an artificial variable Z to get from a 2CNF expression to a 3CNF expression.) (4 points)

Problem 3: Quantum Computing (10 points)

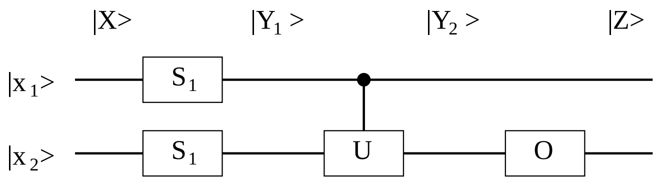

Figure 1: A 2-qubit Quantum circuit.

Consider the apparatus given in Figure 1. Let

$$S_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} e^{i\alpha} & 0 \\ 0 & e^{-i\alpha} \end{pmatrix}$$

and consider an observable O with two subspaces E_1 and E_2. We start with $|X\rangle = |00\rangle$.

(a) Show that S_1 and U are unitary. (2 points)

(b) Compute $|Y_1\rangle$. (2 points)

(c) Compute $|Y_2\rangle$. (2 points)

(e) Let $B_1 = \{|0\rangle\}$ be the basis of E_1 and $B_2 = \{|1\rangle\}$ be the basis of E_2. Compute all possible outcomes for $|Z\rangle$. (Hint: first, separate $|Y_2\rangle$ into two superpositions where one only has $x_2 = 0$ and the other only has $x_2 = 1$.) (4 points)

Problem 4: Dynamical Systems (6 points)

(a) Consider a linear dynamical system with the following transition matrix:

$$P = \begin{pmatrix} 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \\ 2/3 & 1/3 & 0 \end{pmatrix}$$

Show that P is ergodic (i.e. there is a $t > 0$ so that $P^t > 0$) and find the unique stationary stochastic distribution for P. (2 points)

(b) Suggest a sound way to generalize the concept of pairwise mating in quadratic dynamical systems to 3-wise mating. I.e. show that if in your construction the transition matrix is stochastic and the initial distribution $q^{(0)}$ is stochastic, then also $q^{(t)}$ is stochastic for all $t \geq 0$. (Hint: use complete induction.) (4 points)

Good luck!