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# Complexity Theory

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**Final Exam**

**Problem 1: Interactive Proofs** (6 points)

1. Define an IP system and a PCP system, and explain why they are different. (2 points)
2. Suppose that one of the following assertions were true:
   (a) For all classes $\mathcal{F}$ of functions, $\mathrm{PCP}(\mathcal{F},\mathcal{F}) \subseteq \mathrm{IP}(\mathcal{F})$.
   (b) For all classes $\mathcal{F}$ of functions, $\mathrm{IP}(\mathcal{F}) \subseteq \mathrm{PCP}(\mathcal{F},\mathcal{F})$.

   Show that then either P = NP or PSPACE = NEXPTIME. (4 points)

**Problem 2: Approximability** (6 points)
In the MAX NAESAT problem we are given a Boolean expression $\phi$ in CNF, and the problem is to find an assignment for the Boolean variables that maximizes the number of clauses in $\phi$ that have at least one true and one false literal.
Our aim will be to show that MAX NAESAT is MAXSNP-complete. For this we can use as a fact that MAX2SAT is MAXSNP-complete. Then it remains to prove the following items:

1. Show that MAX NAESAT is in MAXSNP. (2 points)
2. Show that there is an L-reduction from MAX2SAT to MAX NAESAT. (3 points)

   (Hint: Expand the clauses in the MAX2SAT expression in a suitable way.)

What consequences does it have for the approximability of MAX NAESAT that MAX NAESAT is MAXSNP-complete? (1 point)

**Problem 3: Quantum Computing** (4 points)
For any unitary $2 \times 2$ matrix $U$, let $\wedge_n(U)$ denote the controlled $U$ transformation based on the logical AND of $n$ qubits. We call any unitary $2 \times 2$ matrix $U$ or matrix of the form $\wedge_1(U)$ a *basic operation*.

1. Show that for any unitary $2 \times 2$ matrix $U$, $\wedge_n(U)$ can be simulated by the network given in Figure 1, where $V^2 = U$. (2 points)
2. Suppose that every conditional not operation on $n$ qubits, $\wedge_n(\sigma_x)$, can be simulated by $O(n)$ basic operations. Show that in this case, for every unitary $2 \times 2$ matrix $U$, $\wedge_n(U)$ can be simulated by $O(n^2)$ basic operations. (2 points)

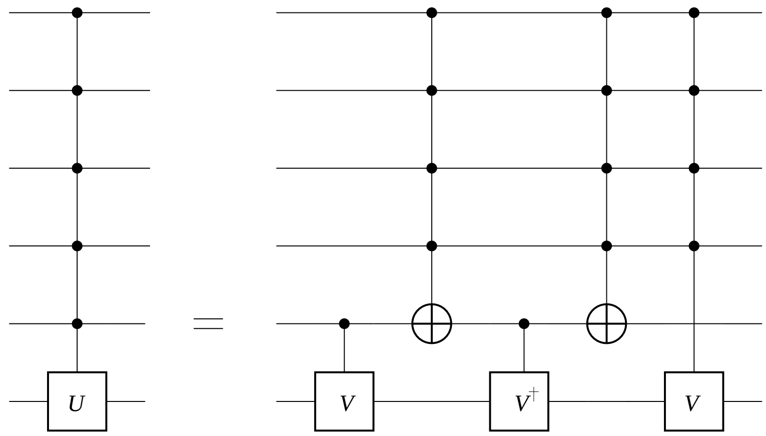


Figure 1: Simulation of $\wedge_n(U)$ for $n = 5$.

**Problem 4: Dynamical Systems** (4 points)

1. Generalize the concept of pairwise mating in quadratic dynamical systems to $k$-wise mating, and use this to define a $k$-ary dynamical system. (1 point)

2. Show that for any fixed $k$, $k$-ary dynamical systems are not more powerful than quadratic dynamical systems. (3 points)

   (Hints: How difficult is it to solve the sampling problem for a $k$-ary dynamical system? How does this compare with quadratic dynamical systems?)

**Good luck!**