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Modern Complexity Theory

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Assignment 9

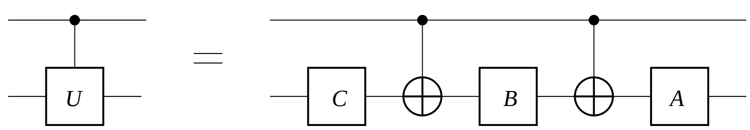
Any *special* unitary 2×2 matrix (i.e., with unit determinant) can be expressed as

$$\begin{pmatrix} e^{i\alpha/2} & 0 \\ 0 & e^{-i\alpha/2} \end{pmatrix} \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \cos(\theta/2) \end{pmatrix} \begin{pmatrix} e^{i\beta/2} & 0 \\ 0 & e^{-i\beta/2} \end{pmatrix} .$$

Consider the following set of standard matrices:

$$R_y(\theta) = \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) \\ -\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}, \quad R_z(\alpha) = \begin{pmatrix} e^{i\alpha/2} & 0 \\ 0 & e^{-i\alpha/2} \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Problem 27 (5 points)**:**
Show that for any special unitary matrix U, the matrices $A = R_z(\alpha) \cdot R_y(\theta/2)$, $B = R_y(-\theta/2) \cdot R_z(-(\alpha+\beta)/2)$, and $C = R_z((\beta-\alpha)/2)$ have the property that $A \cdot B \cdot C = I$ and $A \cdot \sigma_x \cdot B \cdot \sigma_x \cdot C = U$. Use this to show that for any special unitary matrix U there exist unitary matrices A, B, and C such that the controlled U-gate can be simulated by a network of the form



(The vertical line with the $\oplus$ represents a controlled not gate. Recall that $e^{i\delta} = \cos\delta + i\sin\delta$.)

Problem 28 (5 points)**:**
Show that for any unitary 2×2 matrix the U-gate controlled by the logical AND of two qubits can be simulated by a network of the following form with $V^2 = U$.