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Modern Complexity Theory

Spring 2005

Assignment 8

Problem 25 (8 points)**:**
Let $B = \{0, 1\}$ be the set of basis states.

1. Show that $|p\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{2}(1-i)|1\rangle$ is a valid superposition of a quantum computer on B, i.e., $||p||^2 = 1$.

2. Consider the following transition matrix U:

$$U = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} \\ i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

   Show that U is unitary (i.e., that $U^\dagger \cdot U = I$).

3. Compute the outcome $|q\rangle$ of U applied to $|p\rangle$.

4. Compute the amplitude of being in $|v\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{i}{\sqrt{2}}|1\rangle$ given $|q\rangle$ (i.e., compute the complex value of the projection of q onto v).

5. Let $O = \{E_0, E_1\}$ be a standard observation in which event E_0 represents the case that the basis state $|0\rangle$ is observed and E_1 represents the case that the basis state $|1\rangle$ is observed. Compute the probability of E_1 being true given that we are in state $|p\rangle$. Also, compute the new state $|p'\rangle$ that the quantum computer would assume if E_1 is true.

6. Let $O = \{E_0, E_1\}$ be a non-standard observation in which E_0 is represented by the subspace U_0 generated by $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$ and E_1 is represented by the subspace U_1 generated by $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$. Show that this observation still satisfies the conditions of a valid observation (i.e., $U_0 \perp U_1$ and $U_0 \times U_1 = \mathbb{C}^B$). Compute the outcome of $|p\rangle$ when observing E_0.

Problem 26 (2 points)**:**
Show Theorem 10.2.