Dr. Christian Scheideler
Johns Hopkins University

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Modern Complexity Theory

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Assignment 7

Problem 22 (4 points)**:**
In the proof of Theorem 9.1 we showed that if $SAT \in \mathrm{PCP}(log, O(1))$, then MAX3SAT($\epsilon$) is NP-hard for some fixed $\epsilon > 0$. The idea was to transform the computation of the verifier into a 3CNF formula $\phi_{\psi,r}$ so that $\phi_{\psi,r}$ is true if and only if $V^\pi(\psi, r)$ accepts.

1. Suppose that for a given ψ and r, the verifier asks for the bits π_1, π_2, and π_3 in π and only accepts if all of them are equal to 0. Give the 3CNF formula ϕ corresponding to this. (Hint: to simplify this task, establish a truth table for all possible clauses relevant for ϕ; there are eight of them)

2. Under the assumption that there is a PCP for SAT in which the verifier just asks for 3 bits in every computation, find out for which ϵ it is hard to decide MAX3SAT(ϵ).

Problem 23 (2 points)**:**
In the lecture notes, it is claimed after Definition 9.5 that for any graph G, its graph product G^k satisfies

$$w(G^k) = m^k \ \Leftrightarrow \ w(G) = m \ .$$

Prove this statement. (Hint: look at the algorithm in the lecture notes that reduced a clique in G^k to a clique in G.)

Problem 24 (4 points)**:**
The MAXCUT problem is defined as follows: given a graph $G = (V, E)$, find a subset (or *cut*) $U \subseteq V$ so that the number of edges in G with one endpoint in U and one endpoint outside of U is maximized.
The MAX2SAT problem is defined as follows: given a Boolean expression ϕ in 2CNF (every clause has exactly 2 literals), find an assignment that maximizes the number of satisfied clauses in ϕ.
Construct an L-reduction from MAXCUT to MAX2SAT. (Hint: interpret the nodes in V as Boolean variables, and add for each edge $\{u, v\} \in E$ the Boolean expression $(x \vee y) \wedge (\neg x \vee \neg y)$ to $\phi(G)$.)