

**Modern Complexity Theory**  
Spring 2005  
**Assignment 5**

**Problem 16** (4 points):

A graph  $G$  is called *3-colorable* if 3 colors suffice to color its nodes so that no two adjacent nodes have the same color. Consider the problem

$$3\text{COL} = \{\langle G \rangle \mid G \text{ is 3-colorable}\}$$

Show that  $3\text{COL} \in \text{IP}$ . That is, design an interactive proof for  $3\text{COL}$  and show that it satisfies the conditions of IP.

**Problem 17** (3 points):

Prove that NP is equal to the set of all languages  $L$  for which there exists a polynomial time checkable relation  $R_L$  such that

$$L = \{x \mid \exists y (x, y) \in R_L\}$$

and  $(x, y) \in R_L$  only if  $|y| \leq \text{poly}(|x|)$ . Hint: consider the SAT problem.

**Problem 18** (3 points):

Look at the proofs in the lecture notes that  $\#\text{SAT} \in \text{IP}$  and  $\text{PSPACE} \subseteq \text{IP}$  and use the information in these proofs to design an interactive proof for QSAT. (A proof of correctness is not required.)