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Modern Complexity Theory

Spring 2005

Assignment 4

Problem 12 (4 points)**:**

(a) Prove item (9) of Proposition 4.1. (Hint: There are only four different classes of assignments appropriate for ϕ_1 and ϕ_2 that have to be considered. Show that for each of these classes the equivalence holds.)

(b) Prove item (2) of Proposition 4.3. (Hint: There are only two classes of Boolean expressions for ϕ_1 that have to be considered to prove the equivalence.)

Problem 13 (2 points)**:**
Prove via reduction with the help of the language in Problem 7 (or its complement) that the language

$$H = \{\langle M \rangle : \ M \text{ started with an empty input accepts}\}$$

is not recursive.

Problem 14 (3 points)**:**
Complete the proof of the NP-completeness of SAT by deriving logical expressions for "$C_t \rightarrow C_{t+1}$" (2 points) and "$C_t = C_{t+1}$" (1 point). It suffices here to give any suitable polynomial-size Boolean expression for these, i.e. it does not have to be in conjunctive normal form. (Hint: For "$C_t \rightarrow C_{t+1}$" do a $\bigvee$ over all possible locations of the head and express in logical terms that all cells with distance more than one from the head have to keep the same contents and the cells with distance at most one from the head must represent a legal transition.)

Problem 15 (1 point)**:**
Try to formulate the following sentence as a logical expression by defining proper Boolean variables: "Alice uses an umbrella if and only if it rains, and if it rains, Bob will stay at home and watch TV or read a good book."
(Hint: one possible Boolean variable would be A:"Alice uses an umbrella".)