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Modern Complexity Theory

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Assignment 1

Problem 1 (3 points)**:**
Suppose that language $L \subseteq \Sigma^*$ for some finite alphabet Σ has a computer that accepts L. Show that in this case there is also a computer that can enumerate the elements of L (i.e. every element in L has a finite number of predecessors output by that computer).

Hint: Since Σ is finite, all possible words in Σ^* can be enumerated. Let them be $w_0, w_1, w_2, \ldots$. Try to construct a computer that simulates computations of the original computer for L on these words in such a way that any word in $w_0, w_1, w_2, \ldots$ that is accepted by the original computer will eventually be output.

Problem 2 (3 points)**:**
Construct a single-tape DTM that decides the language $L = \{a^n b^{2n} \mid n \in \mathbb{N}\}$.

Problem 3 (4 points)**:**
Show that RP $\subseteq$ BPP and that NP $\subseteq$ PSPACE.

Hint: For the first task, given a language L in RP, take any PTM for L satisfying the RP-conditions and construct out of it a PTM satisfying the BPP-conditions. For the second task, take any problem in NP and use its NTM to construct a deterministic Turing machine for the same problem that simulates the NTM in polynomial space. You do not have to write down Turing machines here. An algorithmic description of the constructions would also be fine.