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Baltimore, April 17 2001

Modern Complexity Theory

Spring 2001

Assignment 10

Any *special* unitary 2×2 matrix (i.e., with unity determinant) can be expressed as

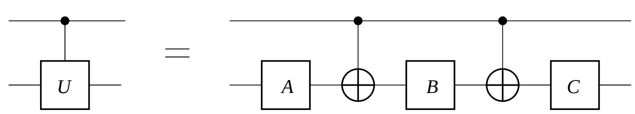
$$\begin{pmatrix} e^{i\alpha/2} & 0 \\ 0 & e^{-i\alpha/2} \end{pmatrix} \begin{pmatrix} \cos(\Theta/2) & \sin(\Theta/2) \\ -\sin(\Theta/2) & \cos(\Theta/2) \end{pmatrix} \begin{pmatrix} e^{i\beta/2} & 0 \\ 0 & e^{-i\beta/2} \end{pmatrix} .$$

Consider the following set of standard matrices:

$$R_y(\Theta) = \begin{pmatrix} \cos(\Theta/2) & \sin(\Theta/2) \\ -\sin(\Theta/2) & \cos(\Theta/2) \end{pmatrix}, \quad R_z(\alpha) = \begin{pmatrix} e^{i\alpha/2} & 0 \\ 0 & e^{-i\alpha/2} \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Problem 29 (2 points)**:**
Show that $\sigma_x \cdot R_y(\Theta) \cdot \sigma_x = R_y(-\Theta)$ and that $\sigma_x \cdot R_z(\alpha) \cdot \sigma_x = R_z(-\alpha)$.

Problem 30 (5 points)**:**
Use the results above to show that for any special unitary matrix U there exist unitary matrices A, B, and C (composed of the standard matrices above) such that the controlled U-gate can be simulated by a network of the form



(The vertical line with the $\oplus$ represents a controlled not gate.)

Problem 31 (3 points)**:**
Show that for any unitary 2×2 matrix the U-gate controlled by the logical AND of two qubits can be simulated by a network of the following form with $V = U^2$.