

## Theory of Network Communication

Fall 2004

### Assignment 5

**Problem 12** (5 points):

Prove that when using the cut-and-paste strategy with  $n$  nodes, the location of every data item can be determined in at most  $O(\log n)$  computations. (3 points)

(Hint: It suffices to show that every data item is replaced at most  $O(\log n)$  times when increasing the system one by one from 1 to  $n$  nodes; replaying these hops gives the time bound for the computations. In order to show that the distances bridged by replacements are essentially increasing exponentially, look at any two consecutive replacements of a data item.)

In addition to this, show that for every perfectly faithful strategy it holds: If the number of nodes grows one by one from 1 to  $n$ , then on average a data item has to be replaced  $\Omega(\log n)$  times. (2 points)

(Hint: compute the total number of data replacements, and divide it by the number of data items.)

**Problem 13** (5 points):

The goal of this problem is to implement a simple form of the consistent hashing strategy in a hypercube. Use as a basis for your implementations the code posted on the course web page for setting up a hypercube and routing in it (or your own code, if you prefer). Extend the code in the following way:

- Implement an operation INSERT that requests as input some positive integer  $x$ . Store  $x$  in the node whose hypercube number  $y$  (which is a number between 0 and  $2^d - 1$ ) has the property that  $y/2^d$  is the closest successor of  $1/x$ . (2 points)
- Implement an operation DELETE that requests as input some positive integer  $x$  and deletes  $x$  from the node that is supposed to store  $x$  according to the INSERT rule above, if it exists. (1 point)
- Implement an operation SEARCH that requests as input some positive number  $x$  and returns as answer either “yes” or “no”, depending on whether  $x$  is stored in the hypercube or not. (2 points)

As usual, attach a printout of the program to your homework and send the program to [scheideler@cs.jhu.edu](mailto:scheideler@cs.jhu.edu).