Dr. Christian Scheideler
Johns Hopkins University

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Theory of Network Communication

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Assignment 2

Problem 3 (1 point)**:**
Show that in the preflow algorithm, the height of every node can be at most $2n - 1$.

Problem 4 (4 points)**:**
Show that for $T \geq n$ the T-balancing algorithm will converge to a fixpoint with $L_{\text{BAL}} = L_{\text{OPT}}$. To do this, revisit the notation in the proof of Theorem 3.13 and suppose that $L_{\text{BAL}} > L_{\text{OPT}}$. Take as granted that in this case there must exist two flow paths p_i and q_i with the same flow value > 0 where $\lambda_{p_i} \leq \lambda_e(P \setminus Q)$ for all edges $e \in p_i$, $\lambda_{q_i} \leq \lambda_e(Q \setminus P)$ for all edges $e \in q_i$, p_i and q_i share the same endpoints, and $\ell_{p_i} > \ell_{q_i}$.
In this case, argue that we can extract two path pieces p'_i and q'_i from p_i and q_i so that p'_i and q'_i are node-disjoint apart from their endpoints and $\ell_{p'_i} > \ell_{q'_i}$. From here, conclude that the average δ_e of an edge in q'_i must be at least $\ell_{p'_i} \cdot T \cdot d/(\ell_{p'_i} - 1)$, and therefore there must exist an edge e in q'_i with $\delta_e \geq (T+1)d$ if $T \geq n$. However, in this case the edge cannot belong to q'_i (see the definition of q_i).

Problem 5 (5 points)**:**
Implement the T-balancing algorithm in the Spheres simulation environment for the following graph.

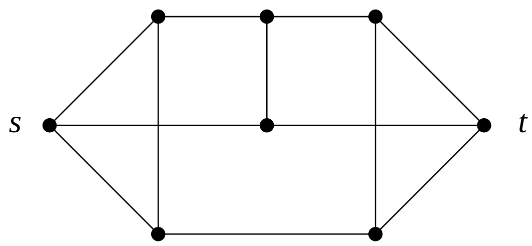

Figure 1: An example graph for the T-balancing algorithm. The source node injects a flow of 2 into the graph at every time step.

Output the flow to which the T-balancing algorithm converges when using the thresholds $T = 2$ and $T = 3$.