Last Minute Bidding and the Rules for Ending Second-Price Auctions: Theory and Evidence from a Natural Experiment on the Internet

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Abstract

There is a great deal of late bidding on internet second price auctions. We show that this need not result from either common value properties of the objects being sold, or irrational behavior: late bidding can occur at equilibrium even in private value auctions. The reason is that very late bids have a positive probability of not being successfully submitted, and this opens a way for bidders to implicitly collude, and avoid bidding wars, in auctions such as those run by eBay, which have a fixed end time. A natural experiment is available because the auctions on Amazon, while operating under otherwise similar rules, do not have a fixed end time, but continue if necessary past the scheduled end time until ten minutes have passed without a bid. The strategic differences in the auction rules are reflected in the auction data by significantly more late bidding on eBay than on Amazon. Furthermore, more experienced bidders on eBay submit late bids more often than do less experienced bidders, while the effect of experience on Amazon goes in the opposite direction. On eBay, there is also more late bidding for antiques than for computers. We also find scale independence in the distribution over time of bidders' last bids, of a form strikingly similar to the 'deadline effect' noted in bargaining: last bids are distributed according to a power law. The evidence suggests that multiple causes contribute to late bidding, with strategic issues related to the rules about ending the auction playing an important role. (*JEL* C73, C90, D44)

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I. Introduction

The recent surge of auctions on the Internet has opened a new window on bidding behavior in second-price auctions, in which the high bidder wins the object being auctioned, but pays a price equal to a small increment above the second highest bid. Internet auctions typically run for several days, and the auction interface allows a bidder to submit a reservation price, and to have this (maximum) price used to bid for him by proxy. That is, a bidder can submit his reservation price (called a proxy bid) early in the auction and have the resulting bid register as the minimum increment above the previous high bid. As subsequent reservation prices are submitted, the bid rises by the minimum increment until the second highest submitted reservation price is exceeded. Hence, an early bid with a reservation price that is higher than any other submitted during the auction will win the auction and pay only the minimum increment above the second highest submitted reservation price. A number of authors, by analogy with sealed bid second-price private value auctions, have suggested that it is a dominant strategy for bidders simply to bid their true reservation price.

However, one does not have to observe an active auction house like eBay for very long before noticing two facts.[1]

1. Many bidders submit multiple bids in the course of the auction (i.e. they submit and later raise the reservation price they authorize for proxy bidding on their behalf); and
2. a non-negligible fraction of bids are submitted in the closing seconds of the auction (a practice called "sniping").

For example, in an exploratory sample of just over 1000 completed eBay auctions sampled in May and June 1999,[2] 28% had 0 bidders, 16% had exactly 1 bidder, and of the remaining 585 auctions, 74% showed multiple bidding (at least one bidder raised his reservation price in the

[1] Indeed, in the course of completing this study and presenting it at seminars we became aware of three interesting and rather different papers at least partly addressed at explaining roughly the same facts: Bajari and Hortaçsu (2000), Malhotra and Murnighan (2000), and Wilcox (2000). Both our methods and many of our conclusions are quite different from theirs, as we will briefly discuss; in particular, we find evidence of multiple causes of these phenomena.

[2] The auctions were sampled from a subset of the different auction categories offered on eBay, in groups of 50 auctions per category.

course of the auction), and 18% had bids in the last sixty seconds. (eBay auctions have a fixed duration set by the seller, typically seven days, and auctions end at the same time of day, to the second, as when they began.) There is substantial variation in the percentage of last-minute bids. The highest percentage in this sample was in the category "Antiques: Ancient World," in which 56% of the auctions that attracted more than one bid had bids in the last sixty seconds, while the lowest such percentage was in "Collectibles: Weird Stuff: Totally Bizarre," where it was 0%.

This behavior arises despite advice from both auctioneers and sellers that bidders should simply submit their maximum willingness to pay, once, early in the auction. For example, eBay instructs bidders on the simple economics of second price auctions, using an example of a winning early bid. And they discuss last minute bids on a page explaining that they will not accept complaints about sniping, as follows:

> "Bid Sniping (last minute bidding)
> eBay always recommends bidding the absolute maximum that one is willing to pay for an item early in the auction. eBay uses a proxy bidding system, you may bid as high as you wish, but the current bid that is registered will only be a small increment above the next lowest bid. The remainder of your Maximum Bid is held, by the system, to be used in the event someone bids against you ... Thus, if one is outbid, one should be at worst, ambivalent toward being outbid. After all, someone else was simply willing to pay more than you wanted to pay for it. If someone does outbid you toward the last minutes of an auction, it may feel unfair, but if you had bid your maximum amount up front and let the Proxy Bidding system work for you, the outcome would not be based on time." [http://pages.ebay.com/aw/notabuse.html, 1999]

Sellers, when urging potential buyers to bid early, are concerned with the fact that very late bids run the risk of not being successfully transmitted (which causes lower expected revenues). The following paragraph on late bidding, posted by a seller, is representative of advice on that subject:

> "THE DANGERS OF LAST MINUTE BIDDING: Almost without fail after an auction has closed we receive emails from bidders who claim they were attempting to place a bid and were unable to get into eBay. There is nothing we can do to help bidders who were "locked out" while trying to place a "last minute" bid. All we can do in this regard is to urge you to place your bids early. If you're serious in your intent to become a winning bidder please avoid eBay's high traffic during the close of an auction. Its certainly your choice how you handle your bidding, but we'd rather see you a winner instead of being left out during the last minute scramble." [Axis Mundi, 1999].

Another warning about late bidding comes from auctionwatch.com, a rich source of information for users of Internet auctions:

> "There are inherent risks in sniping. If you wait too long to bid, the auction could close before your bid is processed. If your maximum doesn't beat the current high bidder, you won't have a second chance to up the ante. And don't overlook the fact that you could be in the company of other snipers who are ready to snipe your snipe. It happens all the time."
> [http://www.auctionwatch.com/awdaily/tipsandtactics/buy-bid2.html, 2000]

Despite all this advice, there is an active exchange of tips in eBay's chat rooms about how to snipe effectively, and even a market for bidding software that makes sniping easy. The following two excerpts from software ads reflect the inclination to bid late:

> "[…] our bidding program BidMaster 2000 provides you complete control. […] Set a bid 7 days ahead, track the item's price during the week, edit your bid time, and amount; when the end of the auction nears WHAM your bid will be placed automatically"

or

> "Dave Eccles saw himself and others being outbid at the last second by 'snipers' and responded by developing a full-blown Windows application, which automatically connects you to the internet, and bids on items you choose, EVEN WHEN YOU SLEEP. It has an intuitive interface which makes it a snap to use - you'll be placing winning auctions with as few as 1 or 2 seconds remaining, even while at work, on vacation or sound asleep."

A number of observers have expressed surprise and puzzlement at this pattern of multiple bidding and last minute bidding. At least one source of the puzzlement at why the "single early bid" strategy is not predominant is the fact that in a second-price, sealed-bid, private-value auction, it is a *dominant* strategy for bidders to submit as their reservation price their maximum willingness to pay. In this view, bidding less than one's true value in a private value auction is an error, and the observed behavior might primarily be due to naïve, inexperienced, or plain irrational behavior (see e.g. Wilcox, 2000, for this view of eBay auctions). Malhotra and Murnighan (2000) also conclude they are seeing irrational late bidding, in a fascinating study of a Chicago charity auction for artists' renderings of cows.

One source of inexperienced behavior might be false analogy with first price auctions. For example, in auctionwatch.com's "tips and tactics for bidders", a strategy called "sentry-bidding" is proposed:

> "Another bidding tactic is sentry bidding--placing a bid and then keeping close watch on it for the duration of the auction. If others try to outbid you, you can quickly place a counter-bid to regain your high bidder status. […] This could lead to an all-out bidding war, so be clear on how much you're willing to spend for an item and how closely you can monitor the auction. And beware of the last-second snipe from your worthy opponent."
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> [http://www.auctionwatch.com/awdaily/tipsandtactics/buy-bid.html, 2000]

Bidders who submit multiple bids—i.e. who raise their bid in the course of the auction—obviously have not submitted their maximum willingness to pay, at least initially. One way to explain the apparent anomaly without positing inexperience or irrationality on the part of the bidders is to note that, if an auction is common-value rather than private-value, bidders can get information from others' bids that causes them to revise their willingness to pay. For example, an interesting paper by Bajari and Hortaçsu (2000) studies these anomalies in eBay auctions of sets of coins in mint condition from the point of view of the common value properties of those auctions. They argue that multiple bidding and the concentration of bids near the deadline are evidence that the auctions they observe must be common value, and claim that this behavior "can not be consistent with the presence of private values" (p. 13).

But while there is no doubt that some auctions have a substantial common value aspect, it is difficult to see how this could apply to many of the auctions in which the anomalous behavior has been observed, e.g. to auctions of commodities for which retail prices are also available on the internet. Landsburg (1999) describes his own multiple-bid and last-minute-bidding behavior on items for which he has a clear private value, and concludes:

> "Maybe eBay just makes me giddy. As a free market aficionado, I am intoxicated by the prospect of one-stop shopping for houses, cars, Beanie Babies, and underwear, all at prices that adjust instantly to the demands of consumers around the globe. Or maybe the behavior of eBayers can be explained only by subtler and more carefully tested theories that have not yet been devised."
>
> Landsburg (1999)

The present paper will show that the observed behavior is consistent with equilibrium, perfectly rational behavior in both public value and private value auctions. Multiple and last-minute bidding behavior can constitute an equilibrium even in a private value second-price auction, in which bidding is conducted over time, with a fixed deadline. The key feature of the model will be the observation, already noted in the comments on the dangers of 'sniping,' that bids placed very near the deadline have some probability of not being successfully transmitted.

There are thus equilibria in which bidders bid late, to avoid a price war that would be set off by early bids. Not only does this show that late bids are consistent with rationality in private value auctions, but the result is quite robust, because late bidding in this way is also a best response when there are naïve bidders who treat the auction like an English first price auction.

Late bidding can also occur for a number of rational reasons in a common value auction. Thus multiple and late bidding cannot, by themselves, be taken as evidence for private or common values, nor for rational or irrational behavior. However we will show that rational late bidding in both common and private value auctions is sensitive to the rules of how the auction ends. This will motivate the empirical part of the study.

We will analyze a natural experiment that arises because of differences in the auction rules of two large auction houses, eBay and Amazon. Auctions conducted by Amazon differ from those conducted by eBay in that, if there is a bid in what would have been the last ten minutes of the auction, then the auction is automatically extended so that it does not end until ten minutes after the final bid. This removes the strategic advantage of holding back until the last minute, so that the late-bidding strategies that are in equilibrium on eBay will not be in equilibrium on Amazon.

We will see that the data on the timing of bids reflect this strategic difference in two ways. First, there is significantly more late bidding on eBay than on Amazon. Second, more experienced bidders on eBay submit late bids more often than do less experienced bidders, while the effect of experience on Amazon goes in the opposite direction.

The data will also reveal a striking scale invariant property of the timing of last bids on both eBay and Amazon. The distribution of bidders' last bids within any end-interval of the auction is the same regardless of the length of the end interval, from e.g. the last ten hours to the last ten minutes, with a high percentage of the bids concentrated at the end of the interval. This implies that the distribution of last bids obeys a power law.

We also conduct a survey of late bidders on eBay that sheds some further light on the models we employ, and the behavior we observe.

II. Last minute bidding in continuous-time second-price auctions

II.1 A strategic model of the eBay bidding environment

We consider here the standard eBay auction format, with a specified minimum opening bid but no (other) seller's reservation price.[3] We will construct a model with a multiplicity of equilibria, including equilibria with last-minute bidding, even in purely private value auctions. For clarity, we first present the strategic structure of the auction, to which we will add the form of players' valuations (private value or common value) when we examine particular equilibria.

The strategic model (the eBay game-form):

- There are n bidders, $N = \{1, \ldots, n\}$.
- There is a minimum initial bid m, and a smallest increment $s > 0$ by which subsequent bids must be raised.
- The "current price" (or "high bid") in an auction with at least two bidders generally equals the minimum increment over the second highest submitted bidders' reservation price. There are two exceptions to this:
  - The price never exceeds the highest submitted reservation price: If the difference between the highest and the second highest submitted reservation price is smaller than the minimum increment, the current price equals the highest reservation price. (If more than one bidder submitted the highest reservation price, the bidder who submitted her bid first is the high bidder at a price equal to the reservation price.)
  - If the current high bidder submits a new, higher reservation price, the current price is not raised, although the number of bids is incremented. [4]

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[3] Sellers typically can choose to set not only a (public) minimum initial bid, but also an additional (secret) reservation price below which they will not sell. We will concentrate here on auctions in which the reservation price is known since bidding in an auction with an unknown reservation price presents additional strategic prospects. In particular, auctions in which the high bid is less than the reserve price sometimes lead to post-sale negotiations. Therefore, there is an incentive for an agent not to bid against himself if he is the current high bidder for fear of going over the reservation price and having to pay it in full.

[4] The latter exception does not appear to be a part of the published rules, but it is readily discovered by experimentation and it is recently confirmed in an informal statement by eBay that is posted in one of eBay's chat rooms [http://remarq.ebay.com/ebay/transcript.asp?g=discuss%2Eebay%2Ebidding&tn=120&sh=fd500cdd6dd92421&idx=-1, 2000]: "If the current high bidder raised his bid, the current bid price wouldn't change because the system won't allow a bidder to bid against himself. So, no matter how many times he bids, the price would stay the same, but the number of bids noted on the bid history page (up under your User ID) will increase." The one exception that

- Each submitted reservation price must exceed both the current high bid and the bidder's last submitted reservation price (i.e. a bidder cannot lower his own previous reservation price).
- The bid history lists the current price and the submission time of each bidder's last submitted reservation price along with the corresponding bidder ID $j \in N$. The highest reservation price is not revealed.
- A player can bid at any time $t \in [0,1) \cup \{1\}$. A player has time to react before the end of the auction to another player's bid at time $t' < 1$, but the reaction cannot be instantaneous, it must be strictly after time t', at an earliest time t_n, such that $t' < t_n < 1$. (For specificity, this earliest reaction times is the first $t_n > t'$, chosen from a countably infinite subset $\{t_n\}$ of [0,1), such that the $\{t_n\}$ converge to 1. The information sets of the game are such that if t is between t_{n-1} and t_n ($t_{n-1} < t < t_n$), then the players know only the bid histories up to t_{n-1}.)[5] That is, in the early part of the bidding, bidders can make their behavior in that half-open interval contingent on other bids observed in the interval—they have time to react to one another.
- At $t = 1$, everyone knows the bid history prior to t, and has time to make exactly one more bid, without knowing what other last minute bids are being placed simultaneously.
- If multiple bids are submitted simultaneously at the same instant t, then they are randomly ordered, and each has equal probability of being received first.
- Bids submitted before time $t = 1$ are successfully transmitted with certainty.
- At time $t = 1$, the probability that a bid is successfully transmitted is $p < 1$. [6]

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is not mentioned in the statement is if the current price is less than the minimum increment above the second highest submitted willingness to pay, which can occur if the high bidder's reservation price was less than the minimum increment above the second highest. In this case, if the high bidder submits a new bid, it will raise the current price to the minimum increment above the second highest reservation price.

[5] This allows us to be sure that the path of play will be well defined for any strategy, and avoid the potential problem with continuous time models of having the path of play undefined, as when each player's strategy calls for him to react to the other players 'immediately.'

[6] It is obviously an approximation both to take a discontinuous drop from $p = 1$ to $p < 1$, and to model the "last minute" as a single point in time. Here we sacrifice strict realism for simplicity and clarity.

II.2 Private value eBay auctions

In a private value auction, each bidder j has a true willingness to pay, v_j, distributed according to some known, bounded distribution F. A bidder who wins the auction at price h earns $v_j - h$, a bidder who does not win earns 0. At $t = 0$ each player j knows her own v_j. At every $t \in (0, 1) \cup \{1\}$, each player knows also the bid histories for all $t_n < t$.

Unlike the case of a sealed-bid second-price auction, a bidder in this continuous-time second-price auction does not have *any* dominant strategy, even in the case of pure private values.

Theorem. A bidder in the continuous-time second-price private value auction does not have any dominant strategies.

Proof. Let $n = 2$ (if $n > 2$ assume that all additional bidders do not bid at any time). It is sufficient to show that there is no strategy for bidder j who has value $v_j > m + s$ that is a best reply to every strategy of the other bidder, i.

Suppose first that i's strategy is to bid the minimum bid m at $t = 0$ and then not to bid further at any information set at which he remains the high bidder, but to bid B (with $B > v_j + s$) whenever he learns that he is not the high bidder. Against this strategy, j's unique best reply is not to bid at any time $t < 1$ (which would provoke a counterbid of B, and cause j to win at most zero), and to bid v_j at time $t = 1$ (at which time the other bidder does not bid, since he does not learn that he is no longer the high bidder until the game is over). The payoff to j from this strategy is $p(v_j - m - s) > 0$, and it is easy to see that no other bid at time $t = 1$ could yield j a larger payoff. (Other bids at $t = 1$, while weakly dominated, would yield the same payoff against i's strategy, and would also be best replies.)

But now suppose instead that player i's strategy is not to bid at any time. Then a strategy for player j that calls for bidding only at $t = 1$ will give j the expected payoff $p(v_j - m - s) < v_j - m$, which is the payoff j could achieve from the strategy of bidding v_j at $t = 0$ (or at any $t < 1$, at which the bid will be transmitted with certainty instead of with probability $p < 1$). Hence no best reply to the strategy of the previous paragraph is among player j's best replies to the strategy for

player i considered in this paragraph, and hence j does not have a dominant strategy, i.e. does not have any strategy that is a best reply to all strategies of the other player.

So the continuous time auction is very different from the sealed bid auction at which every player has a dominant strategy of bidding his maximum willingness to pay. Since there are no dominant strategies in the continuous auction, it is not surprising that there exists a multiplicity of equilibria, including of course equilibria at which each player i bids his true value v_i at $t = 0$. But there are also equilibria at which no player bids his true value until the last moment. The intuition behind last-minute bidding at equilibrium in a private value auction will be that there is an incentive not to bid too high when there is still time for other bidders to react, to avoid a bidding war that will raise the expected final transaction price. And mutual delay until the last minute can raise the expected profit of all bidders, because of the positive probability that another bidder's last-minute bid will not be successfully transmitted. Thus at such an equilibrium, expected bidder profits will be higher (and seller revenue lower) than at the equilibrium at which everyone bids true values early.[7]

Theorem. There can exist equilibria in which bidders do not bid their true values until the last moment, $t = 1$, at which time there is only probability $p < 1$ that the bid will be transmitted.

Proof. It will be sufficient for our purpose to consider the simple case of two bidders, $N = \{1,2\}$, with true values v_1, v_2 each of which are independently, with probability 1/2, equal to either L or H, with $m + s < L < H - s$. Consider the following bidding strategies, which we will show constitute an equilibrium when p, the probability of a successfully transmitted bid at $t = 1$, is not too small compared to $H - L$. On the equilibrium path, each bidder i's strategy is not to bid at any time $t < 1$, unless the other bidder deviates from this strategy. Off the equilibrium path, if player j places a bid at some $t' < 1$, then player i bids v_i at some $t > t'$ such that $t < 1$. That is, each player's strategy is to do nothing until $t = 1$, unless the other bidder makes a bid at some

[7] But the positive probability that a last-minute bid will fail to register also creates a cost, so there is also some incentive to bid early. Thus there can also exist equilibria at which buyers may bid both early and at the very last moment (see Appendix for a proof).

$t < 1$. Any early bid starts a price war at which the equilibrium calls for a player to respond by promptly bidding his true value.

In equilibrium, a player with value L earns $L - m$ if his bid at $t = 1$ is successfully transmitted and the other bidder's bid is lost (which happens with probability $p(1 - p)$). On all other possible equilibrium paths, and whenever the bidder deviates from his equilibrium strategy by bidding at some $t < 1$, his payoff is zero (regardless of whether the opponent's value is H or L and whether bids are lost or not). In the subgame starting at $t = 1$, any other bid than the true value is weakly dominated. This proves that the indicated strategy is a best reply for bidders with value L.

The equilibrium payoff to a player with value H is $H - L - s$ if both bids are successfully transmitted at $t = 1$ (with probability p^2) and the opponent's value is L (with probability 1/2), $H - m$ if his bid is successfully transmitted and the other bidder's bid is lost (with probability $p(1 - p)$), and zero in all other possible equilibrium states (i.e., either if his bid is lost or if both bidders have a value of H and both bids at $t = 1$ are successfully transmitted). If the player deviates and submits a bid at some time instant $t < 1$, his payoff is $H - L - s$ if the other bidder's value is L (with probability 1/2) and zero if the other bidder's value is H (with probability 1/2). This deviation is unprofitable for a player with value H if and only if $\frac{1}{2} p^2(H - L - s) + p(1 - p)(H - m) > \frac{1}{2}(H - L - s)$, or equivalently:

$$\frac{2p}{1+p} > \frac{H - L - s}{H - m},$$

i.e. whenever the probability p of being able to bid successfully at the last moment is not too small. So, since any other bid than the true value at $t = 1$ is weakly dominated, the indicated strategy is also a best reply for bidders with value H if p is large enough. This completes the proof.

The proof shows that even in private value auctions, bidders may have strategic reasons to refrain from bidding true values as long as there is time for others to react, since otherwise they can cause a bidding war that raises the expected transaction price. In the Appendix, we prove that under a more restrictive condition, namely $2p/(1 + 2p) > (H - L - s)/(H - m - s)$, late bidding

can also be accompanied by multiple bidding, i.e. in equilibrium the same bidder may bid at some $t < 1$ and at $t = 1$. Raising one's bid in the course of an auction, as frequently observed in eBay, is not inevitably a sign of irrationality or inexperience on the part of bidders, nor an indication of common value characteristics of the item being auctioned. Multiple bidding can occur at equilibrium of a private value Internet auction with a hard close.

II.3 A common value equilibrium model of late-bidding in eBay

The simplest models of common value auctions treat the bidders as more or less symmetric, except for their private signals about the common value, and sometimes for a private value component. Bidders can use the bids of the other bidders to update their information about the value of the good. In an English auction conducted so that bidders have to stay in the auction to remain active, and can be observed to drop out, the drop out times of other bidders are informative. In an eBay auction, however, only the prices bid so far as well as the bidder identities may be informative, since it cannot be observed if a bidder has dropped out (he might bid again, or for the first time, at $t = 1$). Bajari and Hortaçsu (2000), for example, develop a common value model in which all bidders bid at $t = 1$, to avoid giving other bidders information. (In their model there is no cost to bidding at $t = 1$, however, since bids are transmitted with certainty.)

In general, late bids motivated by information about common values arise either so that bidders can incorporate into their bids the information they have gathered from the earlier bids of others, or so bidders can avoid giving information to others through their own early bids. In eBay auctions, a sharper form of this latter cause of late bidding may arise when there is asymmetric information, and some players are better informed than others. The following example illustrates this kind of equilibrium late bidding. It is motivated by auctions of antiques, in which there may be bidders who are dealer/experts who are better able to identify high value antiques, but who do not themselves have the highest willingness to pay for these once they are identified.

In our simple "*dealer/expert model*", the bidding structure is the same as above, with the seller's reservation price represented by the minimum allowable initial bid, $m > 0$. The object for sale has one of two conditions, "Fake," with probability p_F, or "Genuine," with probability $p_G = 1 - p_F$. There are $n = 2$ (representative) bidders. The first, U, is uninformed but discerning, and

values Genuines more highly than Fakes ($v_U(F) = 0 < v_U(G) = H$), but cannot distinguish them, i.e. cannot tell whether the state of the world is F or G. The second, I, is informed (e.g. an expert dealer), with perfect knowledge of the state of the world, and values $v_I(F) = 0$ and $v_I(G) = H - c$, with $m < H - c + s < H$. Then the common value aspect of this auction arises from the concern of the bidders with whether the object is Genuine. The strategic problem facing the informed bidder is that if the object is Genuine and he reveals this by bidding at any time $t < 1$, then the uninformed bidder has an incentive to outbid him. But the strategic problem facing the uninformed buyer is that, if he bids H without knowing if the object is Genuine, he may find himself losing money by paying $m > 0$ for a Fake.[8]

Theorem. When the probability that the object is Fake is sufficiently high, at any sequentially rational equilibrium in undominated strategies in this "dealer/expert model" the uninformed bidder U does not bid, and the informed bidder bids only if the object is Genuine, in which case he bids $v_I(G) = H - c$ at $t = 1$. If the informed player deviates and makes a positive bid at any $t < 1$, then the uninformed bidder bids H at some t' such that $t < t' < 1$.

Proof. Any strategy that calls for the informed player to bid above his value at any time t is dominated by an otherwise identical strategy that has him bid exactly his value, and hence if the informed player ever makes a positive bid, the uninformed player can conclude that the object is genuine. However so long as $p_F/p_G > (H - m)/m$, it never pays for the uninformed bidder to bid at $t = 1$ if the informed bidder has not already bid, since in this case the expected loss from winning a Fake and paying m is larger than the expected gain from winning a Genuine and paying m. However when no player has bid at $t < 1$, it is a dominant strategy for the informed player to bid his true value when the object is Genuine in the subgame that begins when $t = 1$.

[8] Such a situation is described by esnipe.com as one motivation for why one would want to snipe: "Experienced collectors often find that other bidders watch to see what the experts are bidding on, and then bid on those items themselves. The expert can snipe to prevent other bidders from cashing in on their expertise." (http://esnipe.com/faq.asp)

So in an eBay type auction, an (identifiable) informed bidder has an incentive to not make a high bid on a genuine item until the last minute.[9]

Because there are many ways that values can be interdependent (as opposed to the "pure" or "mineral rights" model of common values), we do not seek here to develop a general model of common values. We simply re-emphasize that the demonstration in the previous section, that late bidding can occur at equilibrium in purely private value auctions, does not imply that late bidding is itself evidence of private values, any more than it is evidence of common values.

Note in particular that the "expertise effect" modeled here that causes experts to bid late can be additive with the "avoiding price war effect" already noted in the private value case. That is, experts could bid late to avoid alerting non-experts of the value of the object, at the same time as they and other bidders bid late to avoid setting off price wars. Section III.3 will show that there is more late bidding in eBay-Antiques than in eBay-Computers. Since these categories might reasonably be expected to have a different scope for expert information, the expertise effect appears to be reflected in the timing of bids.

II.4 A simple model of bidding in Internet auctions with the automatic extension rule

The rules of the auctions on Amazon are like those on eBay except for having no fixed deadline. The relevant Amazon rules are the following:

> "We know that bidding may get hot and heavy near the end of many auctions. Our Going, Going, Gone feature ensures that you always have an opportunity to challenge last-second bids. Here's how it works: whenever a bid is cast in the last 10 minutes of an auction, the auction is automatically extended for an additional 10 minutes from the time of the latest bid. This ensures that an auction can't close until 10 'bidless' minutes have passed. The bottom line? If you're attentive at the end of an auction, you'll always have the opportunity to vie with a new bidder."
>
> [http://auctions.amazon.com/exec/varzea/ts/help/going-going-gone/002-3341436-6525260, 1999]

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[9] The equilibrium is independent of the probability p (with $0 < p < 1$) that a last-minute bid is transmitted successfully since the informed bidder cannot win by bidding early *regardless* of the value of p. Of course, the game has more equilibria, especially if we allow weakly dominated strategies, e.g. there are equilibria at which the informed bidder bids \$1,000,000 + H at $t = 0$ when the object is genuine, and gets the object for m. Note also that we have treated here the case of a single auction. It is not a dominated strategy for a dealer bidding in many auctions to sometimes bid above his value.

Our model of Amazon will be like our eBay model except for the ending rule and, therefore, the times at which bids can be submitted, since Amazon auctions can potentially have arbitrarily many extensions. The strategic significance of Amazon's automatic extension rule is that there is no time at which a bidder can submit a bid to which others will not have an opportunity to respond.

- The times t at which a bid can potentially be made are:[10]

$$[0,1) \cup \{1\} \cup (1,2) \cup \{2\} \cup \ldots \cup (n-1,n) \cup \{n\} \cup \ldots$$

  (As in the eBay model presented earlier, a bidder may *react* to a bid at time t only strictly later than t, i.e. the information sets in each interval are as in the eBay model.)

- The auction extends past any time $\{n\}$ only if at least one bid is successfully transmitted at time $t = n$, and ends at the first n at which no bid is successfully transmitted.[11]

- Bids submitted before time $t = 1$, and in any open interval $(n - 1, n)$, are successfully transmitted with certainty.

- Bids submitted at any $t = n$ for $n = 1,2, \ldots$, are successfully transmitted with $p < 1$.[12]

II.4.1 Private value Amazon auctions

For our private value Amazon model, we assume each bidder j has a true willingness to pay, v_j, distributed according to some known bounded distribution F. Formally, there is some value $v_{\max}$ such that $Pr\{v_j < v_{\max}\} = 1$ for all bidders j. (This will imply that there is an upper bound on how many times an auction can be extended.) We also assume a "willingness to bid" in the case of indifference: A bidder who earns 0 prefers to do it by bidding v_j, and earning $v_j - v_j = 0$,

[10] We model the auction extensions as beginning with open intervals, in order to more cleanly reflect the way Amazon extensions work. However, since we will prove a theorem of the form "no equilibrium exists such that…" it is worth noting that the proof would go through even if we began an auction interval with a semi-closed interval of the form $[n - 1, n)$. That is, the lack of equilibria at which auctions are extended is not an artifact of the fact that there is no earliest possible bid in an extension of the auction.

[11] We model extensions of the auction this way for simplicity: a more realistic model would have auctions extended incrementally further whenever a bid was placed during an extension period.

[12] For simplicity we take the times $t = 1,2,\ldots, n, \ldots$ be both the times in which extensions will be initiated and the times at which bids cannot be transmitted with certainty.

rather than by losing the auction. (Note that this assumption would not have interfered with any of the results presented earlier, for the eBay model). This is a very weak assumption on preferences, since it only comes into play when bidders are indifferent. But since we employ this weak assumption to prove the strong theorem that follows, this is a good point to warn against over-interpreting the theorem. Different reasonable assumptions (e.g. allowing imperfect equilibria) can yield somewhat different conclusions. What the theorem does clearly show is how the automatic extension rule makes late bidding more difficult to achieve at equilibrium, and it rules out the equilibrium we demonstrated for private value eBay style auctions.[13]

Before we state the theorem, it will be helpful to emphasize that in the Amazon auction also, bidders have no dominant strategies. This is a simple corollary of the proof of the same result for the eBay model: the only change needed in the proof is to specify that the strategy considered for the other bidder has him make no bids at $t > 1$.

Corollary. A bidder in the continuous-time second-price private value auction with automatic extension does not have any dominant strategies.

However, the following theorem states that although there are no equilibria in dominated strategies, the sequentially rational equilibria in the continuous second price auction with automatic extension look like the dominant strategy equilibria in a sealed bid auction.

Theorem. At a sequentially rational equilibrium in undominated strategies of an Amazon private value auction, the auction is not extended. All bidders bid their true values before $t = 1$.

Sketch of proof. At a sequentially rational equilibrium in undominated strategies:

1. No bidder ever bids above his value: Any strategy that calls for bidder j to bid above v_j at any time t is dominated by the otherwise identical strategy in which j bids at most v_j at time t.

[13] The fact that different closing rules create different opportunities for collusive behavior has been also observed in the context of the design of the simultaneous ascending first-price auctions that were developed for the sale of radio

2. There is a number n^* such that the auction must end by stage $(n^*-1, n^*) \cup \{n^*\}$, since submitted reservation prices must rise by at least s with each new submission, and (from the previous paragraph) no player will ever submit a reservation price greater than $v_{\max}$. If the auction gets to this last possible stage, there is only room for the price to rise by no more than $2s$.

3. If the auction gets to stage $(n^*-1, n^*) \cup \{n^*\}$, any remaining bidders who are not already the current high bidder and who have a value greater by s than the current price will bid their true value at some $t < n^*$, i.e. at a time when $p = 1$. Recall our willing-bidders assumption—here we use it to rule out possible indifference between bidding and not. Since no bidder is indifferent between casting the winning bid and not, any strategy profile that caused a player to bid at $t = n^*$ would have a lower expected payoff (because $p < 1$) than a strategy at which he bid earlier, when $p = 1$. (And, because this will be the last stage of the auction, the standard Vickrey second-price private value argument implies that a bid of less than the true value would constitute part of a dominated strategy: it could only cause some profitable opportunities to be missed.)

4. Inductive step. Suppose at some stage $(n-1, n) \cup \{n\}$, it is known that at the *next* stage any remaining bidders who are not the current high bidder and who have a value greater than the current price will bid their true value at a time when $p = 1$. Then all bidders will bid their true values before $t = n$. (Since a price war will result if the auction is extended by a successful bid at $t = n$, any strategy profile that calls for a bidder who is not already the high bidder to bid at $t = n$ is not part of an equilibrium, since that bidder gets a higher expected return by bidding his true value at $t < n$.)

5. The auction ends in the first stage: all bidders bid their true value before $t = 1$. (There is also no multiple bidding.)[14]

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spectrum licenses in the United States. See Milgrom (2000) for some related discussion.

[14] Strictly speaking we can conclude all bids are at $t = 0$, but taking this conclusion too literally runs the risk of over interpreting the model. Recall that we assume that one of the bids made at $t = 0$ will be randomly selected to be the first bid received; a sensible way to interpret this is that at the equilibrium bidders bid when they first notice the auction, and this is at a random time after the auction opens.

II.4.2 Common value auctions on Amazon

As already noted, there are many ways in which bidders can have common values, in the sense that one bidder's information conveys information to other bidders about the value to them of the object being auctioned. Some of these could well lead to equilibria in which there is late bidding in Amazon style auctions. However, we will show here that some kinds of late bidding that occur at equilibrium on eBay will not occur at Amazon equilibria. For this purpose, we concentrate on the "expert/dealer" model described earlier.

Theorem. In the "expert/dealer model" (for the one-time auction of a single item), at a sequentially rational equilibrium in undominated strategies, the dealer never wins an auction that uses the automatic extension rule.

Sketch of proof. The dealer values Fakes at 0, and values Genuines less than the uninformed bidder. If the object is Fake, no dealer buys it, since a strategy at which the dealer bids a positive amount for a Fake is dominated by an otherwise identical strategy at which he does not. So if a dealer bids at some time t, he reveals the object is Genuine, and at equilibrium is subsequently outbid with $p = 1$ by the uninformed bidder.

The theorem shows the sense in which Amazon's automatic extension rule robs experts of the ability to fully utilize their expertise without having it exploited by other bidders, by preventing them from "sniping" high quality objects at the last moment. Section III.3 will show that significantly more late bidding is found in antiques auctions than in computer auctions on eBay, but not on Amazon. This suggests that bidding behavior responds to the strategic incentives created by the possession of information, in a way that interacts with the rules of the auction as suggested by our models.

III. A natural experiment

The strategic differences between eBay style, hard close auctions and Amazon style, automatic extension auctions suggest that the following hypotheses about the causes of late bidding can be

investigated by examining the timing of bids on eBay and Amazon. These hypotheses are not mutually exclusive; they could each be contributory causes of late bidding.

	Hypothesis	Predicted contribution to late bidding
Strategic hypotheses	a. *Collusive equilibrium:* bidders bid late to avoid bidding wars with other like-minded bidders.	More late bidding on eBay than on Amazon, with a bigger effect for more experienced bidders.
	b. *Rational response to naïve English auction behavior:* bidders bid late to avoid bidding wars with naïve bidders.[15]	Plus (via part c) more late bidding in categories in which expertise is important than in categories in which it is not.
	c. *Informed bidders protecting their information:* e.g. late bidding by "expert/dealers."	
Non-strategic hypotheses	Bidders bid late because they procrastinate, or because of naïve behavior, or because they don't like to leave bids "hanging," or because search engines present soon-to-expire auctions first, etc.	No difference between eBay and Amazon.

Table 3: Hypotheses about the causes of late bidding

So, the natural experiment will involve looking at the timing of bids in eBay and Amazon, and comparing categories like Antiques and Computers, which might reasonably be expected to have different scope for expert information, between and within the two auction houses.

III.1 Description of the data

Amazon and eBay publicly provide data about the bid history and other features of auctions that have been completed within the last weeks (4 weeks in eBay and 8 weeks in Amazon). We

[15] Note that late bidding may be a best response to a host of "price war" behaviors, of which we have highlighted one prominent behavior. Another such reason to bid late is to foil a seller who attempts (unethically) to bump up the

downloaded data from both auction houses in two categories, "Computers" and "Antiques." In the category Computers, information about the retail price of most items is in general easily available, in particular, because most items are new. The difference between the retail price and each bidder's willingness to pay, however, is private information. In the category Antiques, retail prices are usually not available and the value of an item is often ambiguous and sometimes require experts to appraise.[16] As a consequence, the bids of others are likely to carry information about the item's value, allowing the possibility that experts may wish to conceal their information, as discussed earlier. As we will see, our data support this conjecture.

Our data set consists of randomly selected auctions completed between October 1999 and January 2000 that met certain selection criteria. Auctions were only included if they attracted at least two bidders. Auctions with a hidden reserve price were only considered if the reserve price was met (recall footnote 3). "Dutch Auctions", in which bidders compete for multiple quantities of a single item under different bidding rules than described above, and "Private Auctions", in which the identities and feedback numbers of individual bidders are not revealed, were also excluded.[17]

For the category Computers we selected computer monitors and laptop auctions. For Antiques, we did not restrict our search to a particular subset of items.[18] This is partly to avoid the danger that the data are dominated by atypical behavioral patterns that might have evolved in thin markets for specific antiques, and partly because of a lack of data on Amazon, since relatively few antiques are auctioned there. In total, we selected 480 auctions with 2279 bidders: 120 eBay-Computers with 740 bidders, 120 Amazon-Computers with 595 bidders, 120 eBay-

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price by bidding against you, see e.g. Dobrzynski (2000) in the NY Times for a well publicized example, or Simpson (2000) in the Wall Street Journal for another.

[16] eBay recommends using "appraisal services" such as offered by "The International Society of Appraisers (ISA)" (www.isa-appraisers.org) if one needs help to evaluate an item.

[17] Amazon offers two more auction options: "10% off 1st bidder"-auctions, in which the first bidder locks in a 10% discount from the seller in case of winning, and "Take-it-price"-auctions, in which a bid equal to a seller-defined take-it-price immediately halts the auction and is accepted. 10%-off-1st-bidder-auctions change the incentives for the timing of bidding and were therefore excluded. Take-it-price auctions were only included if the Take-it-price was not met so that the auction ended at the initially posted deadline or, if extended, later.

[18] The items considered here were found in eBay's categories "Computers: Hardware: Monitors" and "Notebooks", respectively, in eBay's category "Antiques: Ancient World", in Amazon's category "Computers & Software" searching for the Keywords "Monitor" and "Laptop", respectively, and in Amazon's category "Arts & Antiques" searching for the Keywords "antiques" or "ancient", respectively. After we collecetd the data, eBay's "Ancient World" category was renamed "Antiquities".

Antiques with 604 bidders, and 120 Amazon-Antiques with 340 bidders.[19] For each auction, we collected data about the number of bids, number of bidders, and about whether there was a reserve price. On the bidder level, we collected information about the "timing" of the last bid and about each bidder's "feedback number". Both variables will be described in detail in the following paragraphs.[20]

Both auction houses provide information about when each bidder's last bid is submitted (but no information about the timing of possible other bids). For each bidder we downloaded the data about how many seconds before the deadline the last bid was submitted if the bid came in within the last 12 hours of the auction time. While this information is readily provided in eBay's bid histories of completed auctions, the end time of an auction in Amazon is endogenously determined since an auction continues past the initially scheduled deadline until ten minutes have passed without a bid. We therefore computed for each last bid in Amazon the number of seconds before a 'hypothetical' deadline. This hypothetical deadline is defined as the current actual deadline at the time of bidding under the assumption that the bid in hand and all subsequent bids were not submitted. Suppose, for example, one bid comes in one minute before the initial closing time and another bidder bids 8 minutes later. Then, the auction is extended by 17 minutes. The first bid therefore is submitted 18 minutes and the second bid 10 minutes before the actual auction close. The bids show up in our data, however, as 60 and 120 seconds (before the hypothetical deadline), respectively.[21]

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[19] eBay maintains a substantially bigger market than Amazon (see Lucking-Reiley, 1999b, for a comprehensive survey of internet auctions, their sizes, revenues, institutions, etc.). For instance, on the supply side, the number of listed items that we found for our Computers-category exceeds Amazon's number in the same time span by a factor of about ten. We finally selected 28% of all randomly drawn eBay auctions and less than 2.5% of all randomly drawn Amazon auctions according to our selection criteria described above. While we found some Dutch, Private, and other auction forms that did not meet our criteria, the large majority of auctions was dropped because of a lack of demand, particularly in Amazon, and because the reserve price was not met.

[20] Actually, we collected more data such as the minimum bid and for each bidder the amount of the last submitted bid. However, since our analysis focuses on the dynamics of bidding rather than on pricing and revenue questions these data will not play a role in our analysis.

[21] Since we only observe the timing of last bids, this calculation implicitly assumes that there is no individual multiple bidding later than 10 minutes before the initial deadline. Suppose, for instance, that the buyer who bids last in the example above also bids 5 minutes before the *initial* closing time. Then, we would have misrepresented the timing of the other buyer's bid, since this bid was then actually submitted 6 minutes before the hypothetical deadline rather than only one minute as computed above. The potential effect of this bias is, however, very small. In total, 28 out of 240 Amazon auctions in our sample were extended. In 26 of these auctions, only one bidder and in the other 2 auctions two bidders bid within the last 10 minutes with respect to the initial deadline. Therefore, we may misrepresent the timing of up to 30 out of 935 Amazon-bidders. Note that the possible misrepresentation of timing

In eBay, buyers and sellers have the opportunity to give each other a positive feedback (+1), a neutral feedback (0), or a negative feedback (– 1) along with a brief comment. A single person can affect a user's feedback number by only one point (even though giving multiple comments on the same participants is possible). The cumulative total of positive and negative feedbacks is what we call the "feedback number" in eBay. It is openly displayed in parentheses just next to the bidder's or seller's eBay-identification name. Amazon provides a related though slightly different reputation mechanism. Buyers and sellers are allowed to post 1-5 star ratings about one another. Both the average number of stars and the cumulative number of ratings are openly displayed just next to the bidder's or seller's Amazon-identification name.[22] We refer to the cumulative number of ratings as the "feedback number" in Amazon. Since in both auction houses the feedback numbers (indirectly) reflect the number of transactions, they might serve as approximations for experience and, more cautiously, as an indicator of expertise.[23]

III.2 Number of bidders per auction and feedback numbers

Different auction categories attract different numbers of bidders with different feedback numbers. Figure 1 shows the cumulative distributions of numbers of bidders across auction houses and categories in our sample. While most auctions had only two bidders (the minimum number that we allowed in our sample), three auctions attracted more than 14 bidders. The most popular item in our data is a Laptop auctioned in eBay with 22 bidders. The figure illustrates that eBay attracts more bidders than Amazon (two-sided χ^2-test, $p = 0.000$) and that within each

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with respect to the hypothetical instead of the actual closing time leads us, if at all, to *overestimate* the extent to which Amazon-bidders bid late. This would only strengthen our comparative results of late bidding in Amazon and eBay.

[22] By clicking on the feedback number in eBay and on the stars in Amazon, everyone has immediate access to the whole history of feedback comments.

[23] Note that the feedback number in eBay is the sum of positive and negative feedbacks. Hence, if positive and negative feedbacks were left with comparable probabilities, the feedback numbers could not be interpreted as experience or expertise. The fact, however, that in our eBay sample *no* bidder (but two sellers) had a negative feedback number while about one third have zero feedback numbers indicates that negative feedbacks are left very rarely. (In fact, eBay encourages users to avoid negative feedback numbers: "If you were treated well by a buyer or seller, reward him or her with a positive comment. If you were treated poorly, try to resolve the problem first by contacting the other person. Most problems can be corrected by improving communication between buyer and seller. If things are still not resolved, you may leave a negative comment.", http://pages.ebay.com/help/basics/f-feedback.html#2, 2000) . Lucking-Reiley et al. (1999) have shown that a seller's feedback number has a measurable effect on the auction price suggesting that sellers' feedback numbers may serve as a measure for reputation.

auction house Computers attract more bidders than Antiques (p = 0.090 for eBay and p = 0.000 for Amazon).

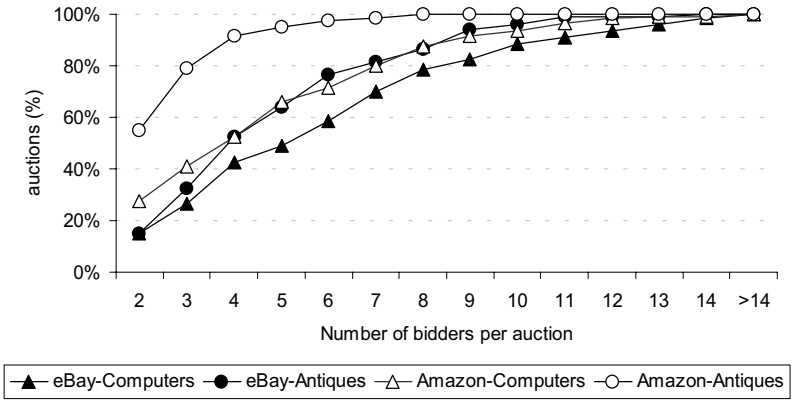

Figure 1: Cumulative distributions of numbers of bidders per auction

There is a great deal of heterogeneity with respect to bidders' feedback numbers in our data. While one third of the bidders have a feedback number of zero, the maximum number is 1162. Figure 2 shows the distributions of Amazon- and eBay-feedback numbers within categories. It illustrates that the numbers are in general higher in eBay than in Amazon (two-sided χ^2-tests based on the categorized feedback numbers as shown in Figure 2 yield p = 0.000 for both, eBay and Amazon separately). Furthermore, there is more bidding activity among antiques-bidders than among computers-bidders within each auction house (p = 0.000 for both, eBay and Amazon separately).

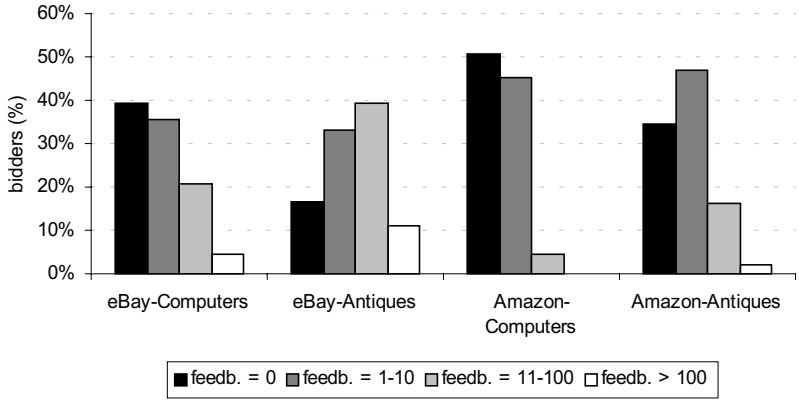

Figure 2: Distributions of feedback numbers

III.3 Late bidding

Figures 3 and 4 illustrate the timing of bids. Figure 3 shows the empirical cumulative probability distributions of the timing of last bids for all bidders and Figure 4 the corresponding graphs for the last bidder in each auction only.[24]

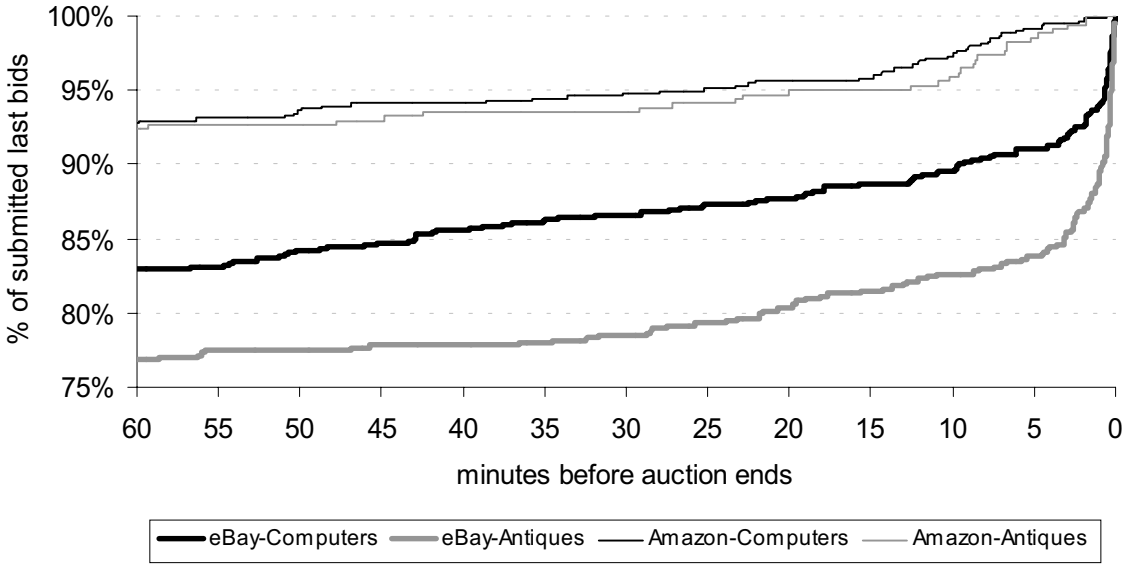

Figure 3: Cumulative distributions over time of bidders' last bids

[24] Recall that the timing of bids in Amazon is defined with respect to a 'hypothetical' deadline that differs from the actual closing time if a bid comes in later than ten minutes before the initial end time. Recall also that the last bidder is not necessarily the high bidder since an earlier submitted proxy bid can outbid subsequently incoming bids. (In total, 156 (6.8%) of all last bids in our sample that are submitted within the last twelve hours are outbid by proxy bidding. At least 32 eBay-bidders were outbid by proxy bidding in the very last minute.) We finally note here that it is not too unusual to see the auction price in eBay double in the last sixty seconds, and since it takes some seconds to make a bid, bidders attempting to submit a bid while the price is rising so rapidly may receive an error message telling them that their bid is under the (current) minimum bid. These eBay-bidders, who attempted to bid in the last minute, are not represented in these data, since their last minute bids did not register as bids in the auction.

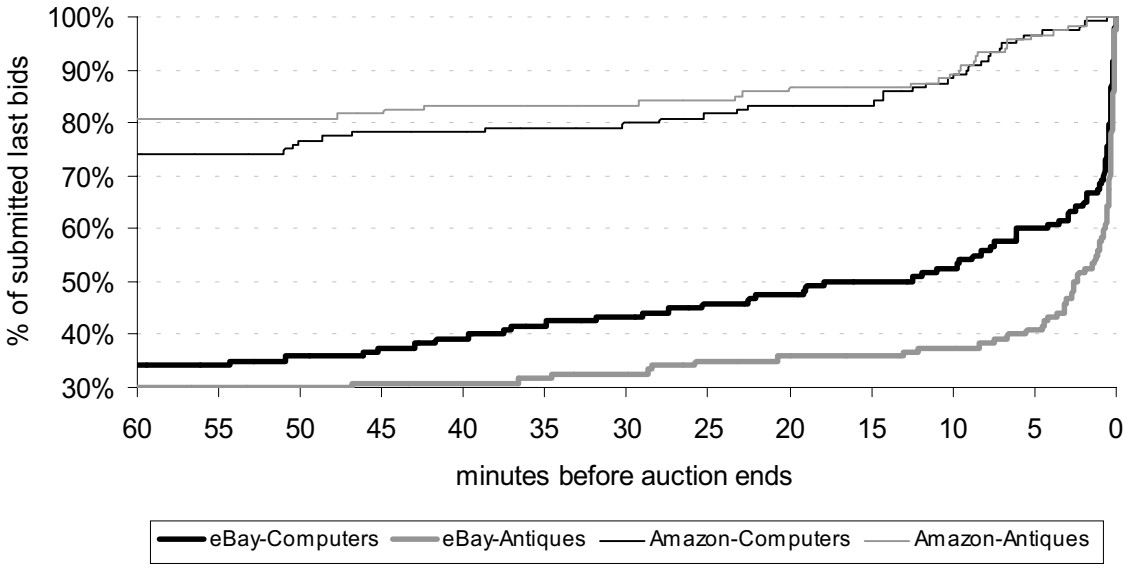

Figure 4: Cumulative distributions over time of auctions' last bids

The figures show that in both auction houses, a considerable share of last bids is submitted in the very last hour of the auctions. (Recall that the auctions usually run for several days.) However, late bidding is substantially more prevalent in eBay than in Amazon. Figure 3 reveals that 20% of all last bids in eBay compared to 7% of all last bids in Amazon were submitted in the last hour. Figure 4 shows that in more than two thirds of all eBay auctions, at least one bidder is still active in the last hour, while this is only true for about one quarter of all Amazon auctions. Furthermore, the figures reveal that also within the last hour, eBay-bids are much more concentrated at the end than Amazon-bids. In particular, in eBay, a considerable share of bidders submit their bid in the last five minutes (9% in Computers and 16% in Antiques), while only few bids come in equally late in Amazon (about 1% in both Computers and Antiques). The difference is even more striking on the auction level: 40% of all eBay-Computers auctions and 59% of all eBay-Antiques auctions as compared to about 3% of both Amazon-Computers and Amazon-Antiques auctions, respectively, have last bids in the last 5 minutes. The pattern repeats in the last minute and even in the last tens seconds. In the 240 eBay-auctions that we downloaded, 89 have bids in the last minute and 29 in the last ten seconds. In Amazon, on the other hand, only one bid arrived in the last minute. The figures also indicate that within eBay,

bidders bid later in Antiques than in Computers. The following regression analyses shed light on how these findings relate to the differences of numbers of bidders and feedback numbers across auction houses and categories.

The dependent variable in the regression analyses in Table 4 is a binary variable with value 1 if the bidder's last bid comes in within the last ten minutes (with respect to the actual or, in Amazon, the hypothetical deadline) and 0 otherwise. The explanatory variables include binary (0/1) variables for the auction house and the item category (*eBay* and *Computers*), as well as the bidders' feedback numbers and the number of active bidders in the corresponding auction (*Feedback#* and *#bidders*). Table 4 reports the maximum likelihood probit coefficient estimates on both the bidder (all bidders) and the auction (last bidders only) level.

The regressions strongly confirm the prediction of the strategic hypotheses that late bidding is more common in eBay than in Amazon; the coefficients for *eBay* are highly significant.[25] eBay auctions attract not only later bids on the bidder level (Run 1), but also the last bid of an eBay-auction has a higher probability of being late (Run 3).

The fuller specifications in Runs 2 and 4 look at interaction effects between auction houses and categories.[26] Within the auction houses, the coefficient estimates in Runs 2 and 4 for *eBay**(1 – *Computers*) are larger than for *eBay***Computers* indicating that eBay-Antiques trigger more late bidding than eBay-Computers. The difference is statistically significant ($p < 0.05$ for Runs 2 and 4 separately). A corresponding test for Amazon auctions yields no significant result ($p = 0.53$ on the bidder level and $p = 0.28$ on the auction level). Across auction houses, on the other hand, eBay attracts highly significantly more late bidding both in Computers and Antiques as well as on the bidder and the auction level ($p < 0.01$ in all four cases) as suggested by Figures 3 and 4.

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[25] The results reported here pass various robustness tests. First, we ran Probit as well as Logit and OLS regressions using 5, 10, and 15 minutes thresholds for late bidding. We found that our qualitative findings are quite insensitive to the statistical model or the threshold for late bidding. (We discuss the full shape of the distributions of last bids in the next section.) Second, in a pilot study, we downloaded data from eBay and Amazon in 320 auctions of computer monitors and antique books. The data set is less complete since only last bidders and only two feedback categories were considered. To the extent we can compare the data with the data reported in this paper, however, they agree in essentially all qualitative features described here.

[26] Runs 2 and 4 both include a constant variable so that we did not include a variable (1 – *eBay*)*(1 – *Computers*) in order to avoid linear dependencies among the explanatory variables.

Explanatory variables	All bidders' last bids		Auctions' last bids	
	Run 1	*Run 2*	*Run 3*	*Run 4*
	coeff. (*p*-value)	coeff. (*p*-value)	coeff. (*p*-value)	coeff. (*p*-value)
Constant	– 1.567 (0.000)	– 1.454 (0.000)	– 1.404 (0.000)	– 1.240 (0.000)
eBay (= 1 if eBay and 0 if Amazon)	0.773 (0.000)		1.140 (0.000)	
Computers (= 1 if Computers and 0 if Antiques)	– 0.187 (0.021)		– 0.260 (0.063)	
Feedback#	0.001 (0.012)		0.003 (0.017)	
#bidders	– 0.036 (0.003)		0.074 (0.001)	
*eBay*Computers*		0.477 (0.026)		0.682 (0.026)
(1 – *eBay*)**Computers*		– 0.264 (0.168)		– 0.268 (0.284)
*eBay**(1 – *Computers*)		0.689 (0.001)		1.033 (0.001)
*eBay*Feedback#*		0.001 (0.008)		0.003 (0.015)
(1 – *eBay*)**Feedback#*		– 0.034 (0.059)		– 0.035 (0.095)
eBay#bidders*		– 0.039 (0.003)		0.071 (0.007)
(1 – *eBay*)**#bidders*		– 0.026 (0.401)		0.068 (0.124)
Log-likelihood	– 650.6	– 646.4	– 239.9	– 236.8
Number of observations	2279	2279	480	480

The dependent variable is 1 for bidders whose last bid came in the last 10 minutes before the (in Amazon: 'hypothetical') deadline and 0 otherwise. The table reports maximum likelihood probit coefficient estimates.

Table 4: Probit-estimates for late bidding in eBay and Amazon

The regressions reveal an interesting correlation between feedback numbers and late bidding. Runs 2 and 4 show that the impact of the feedback number on late bidding is significantly positive in eBay (p = 0.008 for Run 2 and p = 0.015 for Run 4) and negative in Amazon (p = 0.059 for Run 2 and p = 0.095 for Run 4). This suggests that more experienced bidders on eBay go later than less experienced bidders, while experience in Amazon has the opposite effect, as

suggested by the strategic hypotheses.[27] (Wilcox, 2000, examines a sample of eBay auctions and also finds that more experienced bidders bid later.)

The regressions also reveal an impact of the number of bidders on late bidding. Runs 1 and 2 indicate that the number of bidders is negatively correlated with the average tendency of bidding late and Runs 3 and 4 indicate that the number of bidders is positively correlated with the probability that the last bid of an auction arrives late. While Runs 2 and 4 show that this size effect is significant in eBay but not in Amazon, the differences between the eBay- and Amazon-coefficients are not significant ($p = 0.70$ on the bidder level and $p = 0.96$ on the auction level).

The regression analysis confirms the impression given by Figures 3 and 4. There is substantially more last minute bidding in eBay than in Amazon regardless of the category. This difference also grows with the experience of the bidders. It is therefore safe to conclude that last minute bidding is not simply due to naïve time-dependent bidding. Rather, it responds to the strategic structure of the auction rules in a predictable way. In addition, significantly more late bidding is found in antiques auctions than in computer auctions on eBay, but not on Amazon. This suggests that bidding behavior additionally responds to the strategic incentives created by the possession of information, in a way that interacts with the rules of the auction.

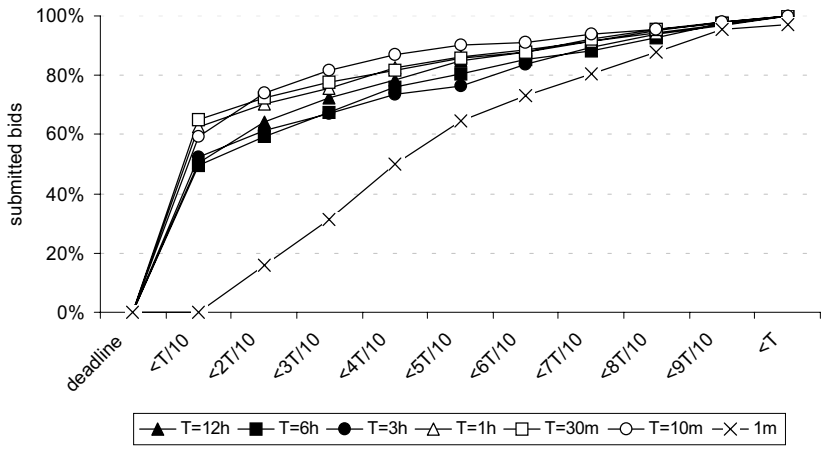
III.4 A closer look at the timing of last bids

The simple strategic model of eBay presented in section II.1 of this paper supposes that there is a unique point of time, such that all bidders can respond to bids that come in before but cannot respond to bids that come in later than this threshold. This abstracts away from the fact that the ability and opportunity to bid late may differ for different bidders. Even if one presumes that all bidders actually want to bid late, the probability that they do so depends on a mixture of technical, physical, and social factors such as weekday, working hours, time zone, physical reaction time, internet congestion, modem speed, etc.

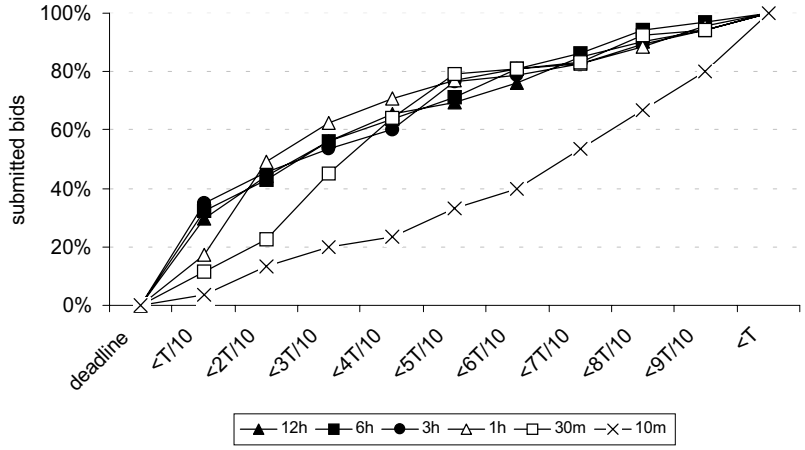
We find a striking regularity in the distribution over time of bidders' last bids: it is almost impossible to say whether a distribution of last bids is drawn from, say, the last hour or from the

[27] The difference between the coefficient estimates is significant on the bidder level ($p < 0.05$) and weakly significant on the auction level ($p = 0.069$).

last 12 hours if no information about the time scale is given. Figure 5 illustrates this regularity. It shows empirical cumulative distribution functions F_T for end intervals $[0,T]$, where 0 represents the auction deadline and T = 12 hours, 6 hours, 3 hours, 1 hour, 30 minutes, 10 minutes, and, for eBay only, 1 minute. In the figures below, each F_T graphs the percentage of last bids received in the interval $[0,T]$ that were received in each decile of the interval. That is, the percentage of bids received in the first decile (T/10) are the percentage of the bids that were recorded in the last 10% of the interval. (Since we do not have data about the distribution of bids that are submitted more than 12 hours before the deadline, we restrict our analysis in the following to bids with $T \leq$ 12 hours.)



(a) eBay



(b) Amazon

Figure 5: Scale independence in the distributions of last bids in eBay and Amazon

While shorter intervals generally induce somewhat more bidding in the first decile in eBay and less bidding in the first decile in Amazon,[28] the probability mass in each decile is surprisingly independent of the length of the corresponding interval with the exception of the shortest interval in eBay and Amazon, respectively. In eBay, averaging over all end intervals with $T > 1$ minute in Figure 5, about 57% of all bids are submitted in the first decile (with a mean deviation of 6%). That is, for any end interval T from 12 hours to 10 minutes, about 57% of the last bids that arrive in the interval arise in the last 10%. The last minute on eBay does not share the distribution common to all the other intervals, and is not concentrated at the end of the interval, perhaps reflecting bidders' attempts to beat the deadline. In Amazon, on the other hand, averaging over all end intervals with $T > 10$ minutes in Figure 5, only about 25% of all bids are submitted in the first decile (with a mean deviation of 8%). Similarly to eBay, the last ten minutes on Amazon are less weighted towards the end than the longer intervals. This is reflected also in the last three deciles of the $T = 30$ minutes distribution (representing the last 3, 6, and 9 minutes) and by the last decile of the $T = 1$ hour distribution (representing the last 6 minutes).

Equation (1) characterizes the scale independence (i.e. independence of T) of the other distributions F_T for end intervals $[0,T]$. It states that the probability that the last bid is submitted in the last at seconds given the auction lasts t more seconds only depends on the percentile (a) but not on the length of the interval (t).

(1) $$\frac{F_T(at)}{F_T(t)} = g(a) \text{ for all } a \in [0,1] \text{ and all } 0 \le t \le T,$$

Equation (1) implies a cumulative distribution function of a power-functional form: [29]

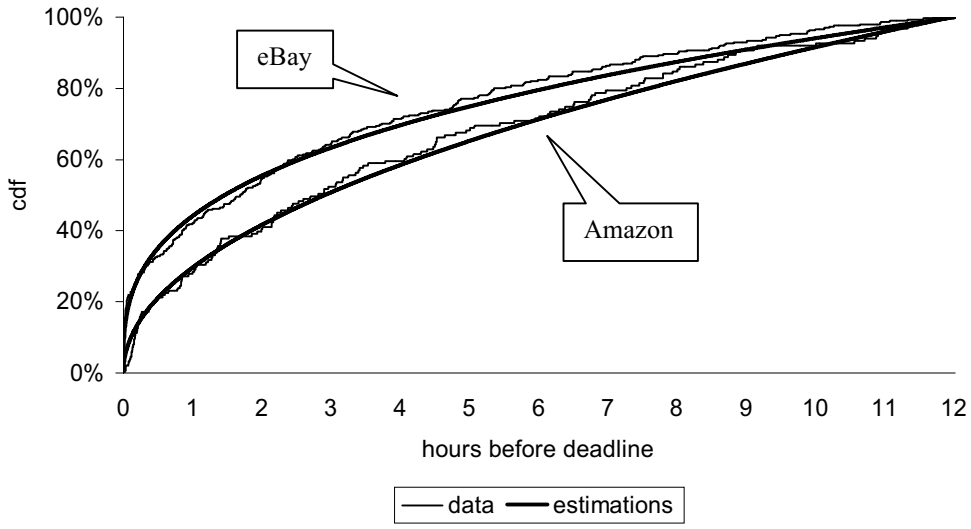
$$F_T(t) = \left(\frac{t}{T}\right)^{\alpha}.$$

[28] Applying χ^2-tests, pairwise comparisons of the F_T graphs reveal that these effects are statistically significant at the 5%-level for eBay and Amazon, respectively.

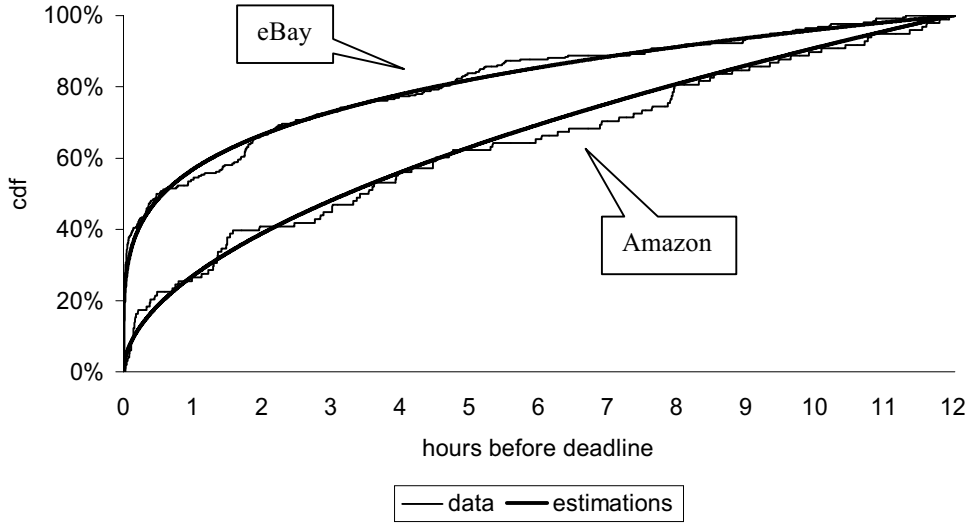
[29] From (1) it follows that $g(ab) = g(b)g(a)$ for all $b \in [0,1]$. Hence, by applying the Cauchy equation (see, e.g. Açzel, 1966, p. 41), $g(a) = a^{\alpha}$ for some α. Setting $t = 1$ in (1) together with $F_T(T) = 1$ then yields the power function.

Ordinary least-square estimates for α using as the dependent variable the logarithm of the cumulative distribution of the sample and as the explanatory variable the logarithm of t/T with T = 12 hours, yield α = 0.392 for eBay Computers (R^2 = 0.99, N = 301) and 0.228 for eBay Antiques (R^2 = 0.99, N = 260). The same method yields α = 0.522 for Amazon Computers (R^2 = 0.97, N = 151) and 0.529 for Amazon Antiques (R^2 = 0.99, N = 98). Based on these coefficient estimates, Figure 6 shows the actual and estimated cumulative distributions.[30]

[30] Note that the slope $\alpha \geq 0$ is a measure of the concentration of bids at the deadline; α = 1 implies a uniform distribution of bids over time and α = 0 implies that all last bids are submitted in the last second. Recall, however, that the timing in Amazon is computed with the help of the hypothetical deadline, which made a difference for 30 Amazon-bidders, and which makes a direct comparison harder.



(a) Computers (eBay: α = 0.392; Amazon: α = 0.522)



(b) Antiques (eBay: α = 0.228; Amazon: α = 0.529)

Figure 6: Estimated cumulative distribution functions $F_T(t) = (t/T)^{\alpha}$ and actual data for the timing of the last bids in eBay and Amazon

Scale independence of this sort has been observed with respect to many economic and social variables, yet completely satisfying explanations for this phenomenon are not always available.[31] Most interestingly, scale independence has also been observed in the distribution of agreements over time in bargaining experiments. As in auction bids, bargaining agreements exhibit a pronounced deadline effect; Roth, Murninghan, and Schoumaker (1988) report a striking concentration of agreements reached in the last seconds before the deadline. (Note that, as in eBay auctions, bargaining with a fixed deadline creates incentives to act very late *because* there is a risk of disagreement associated with waiting too long; see e.g., Fershtman and Seidmann, 1993, and Ma and Manove, 1993). Roth et al. note that all their experimental studies "[…] show a substantial number of late agreements, even as late as the last second. Although the choice of final intervals is arbitrary, there is a clear concentration of agreements near the deadline regardless of the interval chosen to measure 'nearness' to the deadline" (p. 811). Kennan and Wilson (1993, p. 95) further analyzed the experimental data. Although they did not discuss power law distributions, they emphasize the self-similarity that we also observe in the Internet auctions:

> "[…] an increasing settlement rate is an artifact of the way time is measured. For example, Roth, Murninghan, and Schoumaker (1988, Figures 1A and 4A) reported data on 621 agreements in bargaining periods that lasted nine minutes, with 11 percent (69) of these agreements made in the first third of the bargaining period. After three minutes have passed without agreement, the remaining six minutes can be treated as a rescaling of time, with the implication that 11 percent of the remaining agreements should be made in the first third of the remaining time. In fact, the reported data show that 13 percent of the remaining 552

[31] One of the first empirical regularities in economics is Pareto's law. It states that – independent of time, space, and institutions – the number of households N having income greater than x is distributed according to the Pareto distribution $N = Ax^{-\alpha}$. Most explanations for Pareto's law are based on stochastic processes involving variables such as mortality and interest rates (see, for instance, Wold and Whittle, 1957), but the income distribution may also be viewed as the result of individual decision making (see Mandelbrot, 1962; see also Persky, 1992, and the references therein for a detailed account of the history of Pareto's law). Similarly, business firm sizes, that are often found to be distributed according to Pareto distributions, can be generated by stochastic processes (Simon and Bonini, 1958). However, as Simon (1991, p. 29) puts it "Without the introduction of very particular ad hoc assumptions, unbutressed by empirical evidence, neoclassical theory provides no explanation for the repeated appearance of Pareto distributions of business firm sizes in virtually all situations where size distributions have been studied". More recently, it is found that scaling independence also occurs in the Internet: the distribution of links that Internet sites receive from other sites, and the distribution of visitors per site follows a power law (see Huberman et al., 1998, and Adamic and Huberman, 1999 and 2000). See also Zipf (1949) and Mandelbrot (1966) for compilations of a wide range of examples for social, economic, and other phenomena with scaling independence properties.

agreements were concluded in the first third of the last four minutes. This does not explain why the agreement rate in the initial three minutes was 11 percent, but given any initial agreement rate, the rescaling argument explains the deadline effect."

The question why both the timing of bids in Internet auctions and the timing of agreements in bargaining experiments follow a power-law distribution goes beyond the scope of this paper. We only note here, that the empirical regularity might be deduced from stochastic models involving scaling properties of exogenous physical variables. The distribution of last bids may, for instance, reflect the distribution of bidders' availability to bid late. Furthermore, since the price increases over time, which prevents bidders from bidding late if their reservation price has been exceeded, the distribution of last bids over time might also be related to the distribution of individual valuations for the item.

III.5 A note on multiple bidding

In our sample, there are on average 2.6 bids per bidder in Amazon and 1.6 bids per bidder in eBay auctions. Unfortunately, individual data on multiple bidding in completed auctions with more than one bidder is not available since both Amazon and eBay only provide information about the total number of submitted bids. An analysis of multiple bidding therefore has to be restricted to the auction level.[32]

The following equation reports ordinary least-squares coefficients (along with the corresponding *p*-values). The dependent variable is the number of bids per bidder per auction. *Reserve* is a dummy with value 2 if the auction has a (hidden) reserve price and 1 otherwise. All other explanatory variables are defined as above.

$$\begin{aligned} \#bids/bidder = \; & \underset{(0.000)}{1.298} + \underset{(0.2939)}{0.375}*eBay - \underset{(0.004)}{0.318}*Computers + \underset{(0.327)}{0.019}*\#bidders \\ & + \underset{(0.666)}{0.106}*eBay*Reserve + \underset{(0.000)}{1.037}*(1 - eBay)*Reserve \end{aligned}$$

(Adjusted R^2 = 0.201; number of observations: 480)

[32] Yahoo.com, which also offers auction services, provide a fuller bid history during their auctions from which individual bid updating can be completely traced back. Unfortunately, however, bidding histories are not available for completed auctions.

The regression analysis shows that antique items trigger significantly more multiple bidding than computers which is in line with the idea that there is more information revelation over time in Antiques than in Computers. Further, the regression reveals that there is no significant difference between eBay and Amazon and no impact of the auction size. However, an auction with a reserve price triggers more multiple bidding than an auction without reserve price, in particular in Amazon. This might reflect that concealing the willingness to pay is a rational strategy as long as the reserve price is not met, since in this case post-auction negotiations between the seller and the high bidder are not unlikely (see footnote 3).[33]

The private value equilibrium presented in section II.1 has the property that the bidders in eBay bid against themselves at $t = 1$, even if there have not been any other bidders. We frequently observed completed auctions with a single bidder bidding multiple times. In particular, among those randomly drawn auctions that did not meet our selection criteria described in section III.1, 121 eBay and 579 Amazon auctions, that met the reserve price, were not selected because of only one bidder. Among these auctions, we observed a considerable number of multiple bids in eBay (about 10 percent) but relatively few in Amazon (about 1 percent).

III.6 Survey

The analysis of the choice data in the previous sections reveals that there is a strategic component to late bidding. In this section, we complement our analysis with the help of survey data. The survey tells us something about what drives late bidding from the bidders' perspective and provides information about unobservable parameters such as the risk that a late bid fails to be transmitted.

368 eBay-bidders who successfully bid at least once in the last minute of an auction were sent a questionnaire with 8 questions. We included approximately the same number of bidders who bid late in Computers and Antiques. 20% (30 Computers- and 43 Antiques-bidders)

[33] We also aggregated in various ways the feedback number of each auction's bidders, but we could not find any significant effects so we omit the corresponding results here.

responded to our survey. Here, we very briefly report some typical patterns in the answers that directly address issues raised by our theoretical models.[34]

A large majority of responders (91%) confirm that late bidding is typically part of their early planned bidding strategy (Question 1, $N = 65$).[35] Among the 49 bidders who verbally explain why they are bidding late, almost two thirds (65%) unambiguously express that they try to avoid a "bidding war" or to keep the price down. In addition, some experienced Antiques-bidders (about 10% of all responders, mostly with high feedback numbers) explicitly state that late bidding enables them to avoid sharing valuable information with other bidders.[36] At the same time, some bidders say that they are sometimes influenced by the bids and the bidding activity of others, although 88% of the late bidders in our survey say that they have a clear idea, early in the auction, about what they are willing to pay (Question 6, $N = 65$). But besides this supportive evidence for strategic late bidding, we also find some indications of naïve late bidding. A few bidders (less than 10%, mostly with zero feedback number) appear to confuse eBay with an English auction.[37] Also, some bidders sometimes felt regret about not being the high bidder or for being the high bidder (Question 7). The median response for those who gave a quantitative estimation for how often this happened is, however, 0% in either case (N = 48 and 46, respectively).

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[34] See the Appendix for the complete questionnaire and a sample of typical answers to each question. Since we only surveyed late bidders, the sample is not representative of the whole bidder population. For instance, the fact that late bidders tend to be more experienced is also reflected in our survey sample. The average feedback number of bidders in eBay in our choice data is 29 while the average feedback number in our survey data is 83.

[35] Not all bidders answered all questions. The number N in parentheses refers to the number of answers to the corresponding question.

[36] Here are three examples of responses from Antiques-bidders: "I know that certain other parties will always chase my bid." (feedback number = 649); "I do so in part because I have found that when I bid early I tend (nearly always) to be outbid, even if I put in a high bid. MAYBE this is because I am thought to have special knowledge about what is a good item (e.g., due to my books)." (feedback number = 182); "The most difficult part is ascertaining the genuineness of a particular piece. If it is fake then I lost the game and my knowledge was inadequate. This is where it is important not to bid early on an item. If you are well known as an expert and you bid. Then you have authenticated the item for free and invite bidding by others." (feedback number = 47).

[37] One bidder explains his late bidding as follows: "… Because I will then know if the price is low enough for the item." (feedback number = 0), and another bidder writes: "… I would also be sure that other bidders wouldn't outbid me." (feedback number = 0). Interestingly, some more experienced bidders realize that beginners are particularly impatient when bidding: "Many new buyers are particularly aggressive in making sure they are listed as high bidder." (feedback number = 198); "The newbies want only to win and will bid until their money runs out another reason to wait until the last 30".." (feedback number = 43); "If there are first time bidder (0) then it's best to walk away. They will push the price up just to stay the high bidder." (feedback number = 6).

Although most bidders never use sniping software (93%, Question 2, $N = 67$), many operate with several open windows and synchronize their computer clock with eBay time in order to improve their late bidding performance. Nevertheless, when bidding late, 86% of all bidders testify that it happened at least once to them that they started to make a bid, but the auction was closed before the bid was received (Question 3, $N = 65$). But there is another prevalent risk of late bidding: 90% of all bidders say that sometimes, even though they planned to bid late, something came up that prevented them from being available at the end of the auction so that they could not submit a bid as planned (Question 4, $N = 63$). Most bidders gave a quantitative estimation about how often this happened to them. The median response is 10% for both, Question 3 ($N = 43$) and Question 4 ($N = 52$), respectively.

We can conclude from the survey that late bidding is most often part of a planned strategy, even though – or perhaps partly because – bidders know that there is a risk in bidding late. At the same time, the survey reveals some heterogeneity in why later bidders bid late, including 'bidding war'-, and 'expertise'-explanations, and, to a lesser extent, non-strategic explanations.

IV. Conclusions

Multiple and late bidding in Internet auctions has aroused a good deal of attention. The present paper shows, first theoretically and then empirically, that this behavior can have multiple causes.

- The clear difference in the amount of late bidding on eBay and Amazon is strong evidence that rational strategic considerations play a significant role, because eBay's hard close gives more reason to bid late in private value auctions, in common value auctions, and against naïve incremental bidders. This evidence that rational considerations are at work is strengthened by the observation that the difference is even clearer among more experienced bidders.
- The difference between the amount of late bidding for computers and for antiques supports the hypothesis that the bidders respond to the additional strategic incentives for late bidding in markets in which expertise plays a role in appraising values.
- The substantial amount of late bidding observed on Amazon, (even though substantially less than on eBay) suggests that there are also non-strategic causes of late bidding, possibly due

to naivete or other non-rational cause, particularly since the evidence suggests that it is reduced with experience.[38]

Our evidence is not inconsistent with the conclusions of Bajari and Hortaçsu, Malhotra and Murnighan, and Wilcox. The first two of those papers each looks at an auction of a particular commodity under a fixed set of rules and deduces that the late bidding they observe results from a particular cause (common values, and irrational "competitive arousal," respectively). The third paper looks at auctions of several commodities on eBay, and notes that experienced bidders tend to bid later. But because our theoretical framework and empirical design permits us to compare the auctions of dissimilar commodities using the same auction rules, and similar commodities using different auction rules, the common bidding behavior observed in all three studies can be viewed here in a broader perspective, which casts it in a different light.

That multiple causes can lead to the same behavior should not come as a surprise, because auctions present bidders with multi-dimensional strategic problems, and time is a one-dimensional strategic variable. A similar point was made in Roth and Xing (1994), who studied several dozen markets and submarkets (mostly entry level professional labor markets) in which firms seek strategic advantage by making their transactions earlier than their competitors. There too it appears that similar strategic decisions about timing arise from multiple causes. Much the same can be said about the deadline effect in bargaining documented in Roth et al. (1988).

The presence of multiple causes for the same phenomena means that it remains difficult to unambiguously assess the effects of the different auction designs. For a fixed set of bidders for a given, private value object, our results suggest that a second price auction with a hard close will raise less revenue than one with an automatic extension, because late bidding causes some bids to be lost. But our results also suggest that bidders with the expertise to identify valuable objects will prefer auctions with a hard close. So the present evidence does not allow us to suggest

[38] Of course we do not claim to have exhausted the possible strategic and non-strategic causes of multiple and late bidding in the brief list of hypotheses tested in this study. For example, multiple bidding may also arise in part from strategic signaling associated with the formation of bidder rings, or associated with coordinating bids when multiple similar objects are on sale, or from the (non-strategic) increased ease of tracking auctions on which one has already placed a bid due to the kinds of software support offered by the auction house. And late bidding in Amazon auctions can arise rationally to the extent that the last ten minutes is a sufficiently short interval so as to present a reduced probability of successful bidding (see e.g. the answers to question 4 in our survey).

which design should be preferred by sellers, although it suggests that the answer may depend on the kind of good being auctioned.[39]

Now that economists are increasingly being called upon to design a variety of markets (see e.g. Wilson, 2000, Milgrom, 2000, Roth and Peranson, 1999), we will increasingly need to be alert to the fact that small design differences can elicit important differences in behavior, as we have seen here. Because the consequences of design choices may depend on difficult to observe interactions between market participants, or on aspects of the auctions inadequately controlled in natural experiments (in which buyer and sellers select themselves into different auctions), it seems likely that experimentation under controlled conditions in the laboratory will be a valuable complement to the kind of theoretical comparison and empirical study of field data reported here.

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[39] There is also room for the auction rules to have different kinds of an effect; e.g. perhaps the hard close provides

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Appendix

Proof that multiple bidding and late bidding may occur in equilibrium of a private value Internet auction with a hard close.

Theorem. There can exist equilibria in which bidders make multiple bids, and do not bid their true values until the last moment, $t = 1$, at which time there is only probability $p < 1$ that the bid will be transmitted.

Proof. As in the proof in section II.2 we consider the simple case of two bidders, $N = \{1,2\}$, with true values v_1, v_2 each of which are independently, with probability 1/2, equal to either L or H, with $m + s < L$ and $L + s < H$. Consider the following bidding strategies, which we will show constitute an equilibrium if $2p/(1 + 2p) > (H - L - s)/(H - m - s)$.

On the equilibrium path, each bidder i's strategy is to bid the minimum bid, m, at $t = 0$, and to bid v_i at $t = 1$, and not to bid at any other time, unless the other bidder deviates from this strategy. Off the equilibrium path, if player j places a bid at some $0 < t' < 1$ or if the high bid at $t = 0$ is greater than m, then player i bids v_i at some $t > t'$ such that $t < 1$. That is, each player's strategy is to bid m at $t = 0$ and do nothing else until $t = 1$, unless the other bidder makes a higher bid at $t = 0$ and is detected (by having his bid register as second, so that the price at $t = 0$ becomes $m + s$, where s is the minimum increment), or if the other player makes a bid at some $0 < t' < 1$. Either of these deviations starts a price war at which the equilibrium calls for a player to respond by promptly bidding his true value.

On the equilibrium path, the payoffs to player i for each configuration of values v_i, v_j is given by:

	$v_j = L$	$v_j = H$
$v_i = L$	$\frac{1}{2}(1-p)(L-m)$ $+ \frac{1}{2}p(1-p)(L-m-s)$	$\frac{1}{2}(1-p)(L-m)$ $+ \frac{1}{2}p(1-p)(L-m-s)$
$v_i = H$	$\frac{1}{2}[(1-p)(H-m) + p^2(H-L-s)]$ $+ \frac{1}{2}[p(1-p)(H-m-s) + p^2(H-L-s)]$	$\frac{1}{2}(1-p)(H-m)$ $+ \frac{1}{2}p(1-p)(H-m-s)$

Remark: The first 1/2 corresponds to the probability that bidder i is the high bidder at $t = 0$ (when both bidders bid m), the second corresponds to the complementary event. The smallest bid increment is s.

Table 1: Payoffs to player i on the equilibrium path

To show that this can be an equilibrium, we need to consider each possible deviation. The most profitable potential deviation will come from a player i with a high true value, $v_i = H$. Such an i who bids more than m at $t = 0$ will be detected only when his bid is randomly selected to come in second and has to be raised by the minimum increment above the other player's bid of m.[40] Once detected, i's expected payoff is $\frac{1}{2}(H - L - s) + \frac{1}{2}(H - H) = \frac{1}{2}(H - L - s)$. To calculate the values of p for which this is not a profitable deviation, consider i's payoff when he deviates by bidding H at $t = 0$. To follow the proof it may help to look at the payoffs arrayed in Table 2, which compares the equilibrium payoff to the payoff from deviating in this way in each of the 16 states of the world faced by a player i with value H. These states of the world are determined by the true value of the other bidder, by whether player i's bid at time $t = 0$ will be first or second, by whether player i's bid at $t = 1$ will be successful, and by whether the other player's bid at $t = 1$ will be successful.

Compared to equilibrium play, a bidder i with $v_i = H$ strictly gains from his deviation bid of H at $t = 0$ in the state of the world at which he is first at time $t = 0$ (so that the deviation cannot be detected), $v_j = L$, and the other bidder j successfully submits a bid at $t = 1$ but (had he played the equilibrium) i would have been unsuccessful at $t = 1$. In this case the deviating bidder i with

[40] If the bid is randomly selected to come in first, the other player's bid would not be processed since it does not exceed the current price of m.

$v_i = H$ earns $H - L - s$ instead of 0, so his expected gain from this state of the world is ¼ $(1 - p)p(H - L - s)$. Bidder i with $v_i = H$ similarly gains in the states of the world in which his bid is second at $t = 0$ (so that deviation is detected) if he would not have been successful at $t = 1$, and if the other bidder has a low value $v_j = L$; the expected gain due to this combination of events is $\frac{1}{4}(1 - p)(H - L - s)$.

		$v_j = L$ (probability = ½)				$v_j = H$ (probability = ½)			
		$p(1-p)$	pp	$(1-p)(1-p)$	$(1-p)p$	$p(1-p)$	pp	$(1-p)(1-p)$	$(1-p)p$
H is first at $t = 0$ (defection undetected; prob. = ½)	equilibrium	$H - m$	$H - L - s$	$H - m$	0	$H - m$	0	$H - m$	0
	Defect by bidding H at $t = 0$	$H - m$	$H - L - s$	$H - m$	$H - L - s$	$H - m$	0	$H - m$	0
	Defect payoff minus equilibrium payoff	0	0	0	$\boldsymbol{H - L - s}$	0	0	0	0
H is second at $t = 0$ (defection detected; prob. = ½)	equilibrium	$H - m - s$	$H - L - s$	0	0	$H - m - s$	0	0	0
	Defect by bidding H at $t = 0$	$H - L - s$	$H - L - s$	$H - L - s$	$H - L - s$	0	0	0	0
	Defect payoff minus equilibrium payoff	$\boldsymbol{H - L - s - (H - m - s)}$	0	$\boldsymbol{H - L - s}$	$\boldsymbol{H - L - s}$	$\boldsymbol{-(H - m - s)}$	0	0	0

Gains from defecting = $[\frac{1}{4}(1-p)p + \frac{1}{4}(1-p)p + \frac{1}{4}(1-p)(1-p)][H - L - s] = \frac{1}{4}(1-p)(1+p)[H - L - s]$

Losses from defecting = $\frac{1}{4}(p)(1-p)[H - L - s - 2(H - m - s)]$

Gains + Losses = $\frac{1}{4}(1-p)[(1+p)(H - L - s) + p(H - L - s - 2(H - m - s))]$

Gains + Losses < 0 if and only if π(defect) – π(equilibrium) < 0 if and only if $2p/(1 + 2p) > (H - L - s)/(H - m - s)$

Table 2: Payoffs in case a bidder i with $v_i = H$ deviates from the equilibrium by bidding H at $t = 0$

But bidder i with $v_i = H$ strictly loses compared to equilibrium play in two states of the world, both of which involve his bid coming in second at $t = 0$, so that the deviation is detected. The first of these is when $v_j = L$ and the other player would have been *un*successful at $t = 1$, but i would have been successful. In this case the deviation earns $H - L - s$ while equilibrium play would have earned $H - m - s$. The second loss state when deviation is detected is when $v_j = H$ and the other bidder j would have been unsuccessful at $t = 1$ while i would have been successful. In this case the deviation earns $H - H = 0$, while equilibrium play would have earned $H - m - s$. Both these states occur with probability $\frac{1}{4} p(1 - p)$, so the potential loss is:

$$\tfrac{1}{4} p(1-p)\{[(H-L-s)-(H-m-s)]+[0-(H-m-s)]\}$$

$$= \tfrac{1}{4} p(1-p)[(H-L-s)-2(H-m-s)].$$

Thus this kind of deviation is unprofitable so long as the payoff from deviating is less than the payoff from equilibrium, which occurs if and only if

$$\frac{2p}{1+2p} > \frac{H-L-s}{H-m-s} \tag{1}$$

Regarding other possible deviations from equilibrium, a bidder i with $v_i = H$ who bids more than m at some $0 < t < 1$ is detected for sure and gets $\frac{1}{2}(H - L - s)$. If condition (1) holds, this deviation is unprofitable. Suppose bidder i with value H is not the high bidder at $t = 0$. Then, bidding H at some $0 < t < 1$ is unprofitable if and only if

$$p^2 \frac{1}{2}(H-L-s) + p(1-p)(H-m-s) > \frac{1}{2}(H-L-s),$$

or equivalently

$$\frac{2p}{1+p} > \frac{H-L-s}{H-m-s} \tag{2}$$

Note that whenever condition (1) holds condition (2) holds too. So if a deviation at $t = 0$ is unprofitable for a bidder with value H, a deviation at some $0 < t < 1$, if the other bidder is the high bidder at $t = 0$, is also unprofitable. Now suppose bidder i with value H is the high bidder at $t = 0$. Then, bidding at some $0 < t < 1$ is even less profitable since it additionally eliminates the potential gain that comes when both bids at $t = 1$ are lost.

A bidder i with $v_i = L$ who bids more than m at $t = 0$ will get 0 with probability ½ (when he is detected by a $t = 0$ price greater than m) and will get $(1 - p)(L - m)$ if not detected, for an overall payoff of ½ $(1 - p)(L - m)$. This is an unprofitable deviation, since it yields the same payoff in equilibrium in the case that L is first at $t = 0$, but less otherwise. A bidder i with $v_i = L$ who bids more than m at $0 < t < 1$ is detected for sure, and gets 0.

A bidder i with $v_i = H$ who does not bid at $t = 0$ but only bids at t=1 will get ½ $[p(1 - p)(H - m - s) + p^2(H - L - s)]$ + ½ $(p)(1 - p)(H - m - s)$ (according to whether the other player is an L or an H, each with probability ½). But each of these terms is just what he gets if he does bid at $t = 0$ but is not the first (high) bidder and has to bid again, against an L or H, respectively. So he certainly does better by bidding at $t = 0$, since then he would get the ½ $(1 - p)$ term corresponding to winning at $t = 0$ and not having to face a successful bid.

A bidder i with $v_i = L$ who does not bid early, but only bids at $t = 1$, will get $p(1 - p)(L - m - s)$, which equals what he would have gotten conditional on bidding early and not being the high bidder. This is strictly less than what he gets by playing his equilibrium strategy. (It is easy to check by comparing condition (1) and the sufficient condition for equilibrium sniping derived in section II.2, however, that if condition (1) holds there exist also equilibria in which bidders bid late but not early.)

Thus the indicated strategies constitute an equilibrium whenever condition (1) holds. This completes the proof.

Survey-text

"Hi: We are economists studying bidding on eBay (and sometimes eBay bidder ourselves, as aer51 and aockenfels). We are particularly interested in late bidding, and we are writing to you because we noticed that you have been successful at bidding in the last moment of an auction you participated in.

If you are willing to answer the few questions below, we would be grateful. (If this is an intrusion, however, please simply ignore this message, and accept our apologies.) The easiest way is probably just to hit reply, and intersperse your answers after the questions. We will be careful to keep confidential the source of the answers we get in this study.

1. Do you sometimes plan, early in an auction, to submit a bid at the last minute? If so, why? (Or do you bid as the spirit moves you throughout the auction, and only bid at the last minute if you are outbid near the end?)

2. Do you bid by hand, or do you use bidding software? If by hand, do you simply use the eBay bidder screens, or do you do something more elaborate, like open multiple windows so that you can follow the bidding while preparing a bid?

3. When bidding late, about what percentage of the time would you say it happened that you started to make a bid, but the auction closed before your bid was received?

4. When you have planned to bid late, about what percentage of the time would you say it happened that something came up, so that you were not able to submit a bid as you planned? (We are thinking here of something that prevented you from being available, or from remembering to bid, not the case in which the price had already risen above what you were willing to pay.)

5. On average, would you say you submit only one bid per auction; no more than two bids, or more than two bids?

6. Do you have a good idea, early in the auction, of the maximum you would be willing to pay, or do you often adjust what you are willing to pay based on the bids you see by other bidders? In this respect, are some other particular bidders especially influential, or is it just the general price level and number of bidders that influences your bid?

7. About what percentage of the time, when you are not the high bidder in an auction, do you regret that you did not bid higher? And in about what percentage of the auctions in which you are the high bidder do you regret having bid as high as you did?

8. Is there anything else about your bidding experience that you think it would be helpful for us to know, as we try to understand what's going on?

Many thanks for your help,

Al Roth
http://www.economics.harvard.edu/~aroth/alroth.html

Axel Ockenfels
http://www.uni-magdeburg.de/vwl3/axel.html"

Typical responses

Section III.6 summarizes the main results of our survey and quotes some answers. Here, we show some additional typical and short excerpts of the usually verbal and sometimes very detailed responses to the survey-questions. The excerpts are ordered according to the feedback numbers (in parentheses) of the corresponding responders.

Answers to Question 1:

- "I plan on the last minute if possible to see what is going on in the bid. Isn't that the whole idea in being involved in an auction? I have heard otherwise, but my feeling is I can beat any other bidders to the finish line by waiting to the end." (Computer, 0)
- "I always bid at the last moment ,why?, to get the stuff with out getting in a bidding war." (Computers, 11)
- "I usually plan early if I see the item as it keeps the bidding lower." (Antiques, 45)
- "If I bid early, I have the feeling someone will bid multiple times with small incremental bids just to get above my maximum bid …" (Computers, 54)
- "My strategy for the past 6 months or so has been to bid only in the auction's last 30 seconds (unless I am not able to be online at auction's end, in which case I leave a maximum bid and hope for the best). This policy doesn't raise the price ahead of time and preserves my anonymity. (Computers, 88)
- "Yes. We are trying to keep from driving up the price." (Antiques, 91)
- "I always bid at the last moments, I plan that on every auction. (…) I do not want to get in a bidding war on an auction." (Computers, 101)
- "Yes I plan on late bidding; because that is when you win the auction-early bids only raise the price ... " (Computers, 102)
- "I always bid at the last possible minute. You have a better chance of getting a lower bid." (Antiques, 106)
- "Because others look at my bids and bid because I do." (Antiques, 277)

Answers to Question 2:

- "Strictly by hand." (Computers, 2)
- "I bid by hand with the standard EBay bidding screen, in a single window. I'm a fairly fast, touch-typist, and I have a cable modem. (Frankly, I hadn't thought of opening a second window. Hmmm, I could probably wait until the last 10-15 seconds if I did... Thanks for the tip!)" (Antiques, 3)
- "I do however employ some 'analog tecniques'. Clocks, timers and 'to the second' time synchronization with Ebay." (Antiques, 43)
- "Strictly by hand. I will often bring up two windows, one holding my bid and one showing the current status of the auction." (Antiques, 52)

- “I do it the right way, by hand, no tricks, except the new ‘sign in’ feature on Ebay. That way i don’t have to worry about screwing up my password at the last second.” (Computers, 101)
- “I bid by hand; opening a time screen that is accurate to the second.” (Computers, 102)

Answers to Question 3:

- “I'd say it's happened to me 2 or 3 times, maybe 10% of the number of auctions I've bid seriously on.” (Computers, 5)
- “20” (Computers, 11)
- “Less than 10% so far, dial up 33.6 modem, 45 second enter before end.” (Computers, 17)
- “At first I would miss a few items by waiting. I have changed from a dial up services to cable service and this decreases the delay in placing a bid, therefore I seldom miss a bid that I'm set on getting.” (Antiques, 11)
- “40%” (Antiques, 28)
- “After a couple of early failures, *never*. I have executed 20 successful "30 sec bids" Exception..During peak hours, to check transaction times, I will make several dry runs to determine process times ...” (Antiques, 43)
- “10%-15% of the time, if my ISP is slow.” (Antiques, 88)
- “5%” (Antiques, 295)
- “Almost never. We have a cable modem and this fast access generally allows us, by reloading several times in the last few minutes, to determine about when the final bid needs to be keyed in to make it before auction closing.” (Antiques, 622)

Answers to Question 4:

- “About 7%. Sometimes I do have a life away from my computer :-)” (Computers, 2)
- “Not very often - I set the alarm :) (usually 3am here)” (Antiques, 3)
- “If I was planning a last minute bid but family, friends or other conflicts with end of auction time I will enter my max early or never bid. If I plain forgot to bid I take that as a personal indication of how much I need the item.” (Computers, 17)
- “When I'm not going to be available, I either bid my "maximum" ahead of time or try to get the wife to bid, if she is able.” (Antiques, 30)
- “Excellent question! Probably at least 15-20% of the time. Since I have a life beyond ebay (although my wife may disagree), I sit in on the auctions when I am able…” (Antiques, 43)
- “Zero percentage. I plan ahead. Only an electrical power outage would stop me.” (Antiques, 87)
- “30%” (Antiques, 649)

Answers to Question 5:

- “1- 80,2-10,3-10” (Antiques, 49)
- “I only leave enough time to make one bid, no more.” (Computers, 107)

- “Maybe the first bid, definitely the last, any "midauction" bids makes no sense as it only raises the price.” (Antiques, 108)
- “Generally two. I try to be ethical in my bidding. I wouldn't put in a last minute bid on an item without having previously bid at some point. Let the other folks know there is the potential for competition, at least. Then it's everyone for himself!!” (Antiques, 160)

Answers to Question 6:

- “I know the maximum I am willing to pay. If the auction is no where near my maximum, you can be sure I'll try for it. I have been watching computer/computer related products for ~ 7 years & know what is or isn't a bargain.” (Computers, 0)
- “I always have known what my max bid will be before I place any bid. I will study other similar items on completed auctions, new product prices, or other sources (e.g. Kelly Blue Book values for a vehicle), and determine ahead of time exactly what I am willing to pay.” (Antiques, 3)
- “If it is an item that I am particularly interested in winning, I ALWAYS determine my maximum bid ahead of time, and generally bid that maximum near the end of the auction …” (Antiques, 30)
- “Absolutely! I know the maximum, having already researched previous winning bids versus the purchase price if I were to buy the item new ...” (Antiques, 43)
- “I usually have a max I will bid on and item I'm not influenced by # of bidders. I am influenced by feedback.” (Antiques, 49)
- “I research wholesale and retail prices at such web sites as pricewatch.com., cnet.com, buycomp.com, (ebay also), computers4sure.com, I know well ahead of time if I am shopping for something. I do impulse shop at times, but again I try to stay within boundaries.” (Computer, 96)
- “The second i look at an item, I know the value, and what i am willing to pay.” (Computers, 101)
- “… I determine the value of the item when I see it for the first time and write it down. I then review my list to determine what I want to bid on and place my bids in advance to be entered at the close of the auction…” (Antiques, 198)
- “I generally know what I will pay. Only occasionally do I change my mind - and this often depends on other bidders whom I know have good knowledge of the values of items on which I may place a bid.” (Antiques, 622)

Answers to Question 7:

- “I always have a set price in mind and I am a patient man. If someone outbids me then it was more than I wanted to pay - no regrets. If I am the high bidder then it was worth it - no regrets.” (Computers, 0)
- “I may have some regrets about not spending more time with my wife, but never about an auction. I think I keep things in perspective.” (Computers, 2)
- “I don’t really bother, I know my max and if I get the item with my max or less is fine if not I already have the alternative ready.” (Computers, 23)

- “I have regrets about 30% of the time for not bidding higher. I usually only regret bidding too high when the merchandise is not as described in the auction.” (Antiques, 54)
- “Almost never. Never.” (Antiques, 622)
- “0%” (Antiques, 649)

Answers to Question 8:

- “Profit is the driving force in auctions. Winning at the game, Out maneuvering an opponent, Being just a little more 'gutsy', Holding out until the last possible second thus risking to lose, these are the adrenaline & testosterone of the auction experience.” (Computers, 2)
- “Do you have any type of research grant or funding? Would you be interested in purchasing my step by step method?” (Antiques, 7)
- “If you want to win auctions, bid odd amounts, like $26.51 instead of $25. Most new people bid right on the increment, and sometimes you can beat them by less than the bid increment by doing this.” (Computers, 28)
- “What I have learned would take far too long to put into an email, but here are a couple of hot button issues: - people who charge exhorbitant shipping/handling fees, - sellers of obvious fakes, - sellers posting crap in inappropriate categories, - sellers using a large network of shills....and bragging about it.” (Computers, 33)
- “It ain't just numbers! Perhaps a member of your team should include a psychologist?” (Computers, 43)
- “Interesting subject! It comes up quite frequently on a coin collector's group that I co-moderate, and I was one of the few initially who would publicly defend the "snipers" against the angry outbursts of those who lost a coin in the last few seconds.” (Computers, 52)
- “I sometimes bid early on an item but at a low price just to establish that I have found the item first (I have a couple of friends who collect similar things, we have a rule that whomever bids first can "go for it" so we don't get into a bidding war amongst ourselves...but only if I am really interested in going for it)” (Computers, 54)
- “If I want to buy something on Ebay, I plan my life around the last five minutes of an auction – I rearrange lunch time, I change my evening plans, I get up early in the morning, whatever it takes to BUY the item I want.” (Antiques, 87)
- “I hate to give away my strategy because I have been very successful.” (Computers, 136)
- “Looking at bid amounts that are odd and cross checking with bid histories will sometime reveal if the bidders bid is at it's max amount.” (Antiques, 206)