Novel Reconstruction and Feature Exploitation Techniques for Sensorless Freehand 3D Ultrasound

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ABSTRACT

Out-of-plane motion in freehand 3D ultrasound can be estimated using the correlation of corresponding patches, leading to sensorless freehand 3D ultrasound systems. The correlation between two images is related to their distance by calibrating the ultrasound probe: the probe is moved with an accurate stage (or with a robot in this work) and images of a phantom are collected, such that the position of each image is known. Since parts of the calibration curve with higher derivative give lower displacement estimation error, previous work limits displacement estimation to parts with maximum derivative. In this paper, we first propose a novel method for exploiting the entire calibration curve by using a maximum likelihood estimator (MLE). We then propose for the first time using constrains inside the image to enhance the accuracy of out-of-plane motion estimation. We specifically use continuity constraint of a needle to reduce the variance of the estimated out-of-plane motion. Simulation and real tissue experimental results are presented.

Keywords: 3D ultrasound, Speckle decorrelation, Fully developed speckle

1. INTRODUCTION

Most common techniques for acquiring 3D ultrasound data are oscillating head probes and freehand 3D ultrasound. In oscillating head probes, a 1D ultrasound transducer is automatically swept inside the probe, enabling 3D image acquisition. In freehand 3D ultrasound, a position sensor is attached to an ordinary probe which is swept over the desire region by the clinician.

Freehand 3D ultrasound is inexpensive, works with the existing 2D probes, and allows arbitrary 3D volume acquisition. However, the need for the additional sensor makes it difficult to use. Sensorless volume reconstruction of freehand 3D ultrasound is possible using the information in the images themselves: out of plane motion estimation can be obtained from image correlation\textsuperscript{,1} which is the focus of this work, while in plane motion can be estimated through image registration\textsuperscript{2–4} or by using techniques similar to elastography\textsuperscript{5–7}.

The granular appearance of ultrasound images is the key factor in out-of-plane motion estimation (Figure 1). Each pixel in an ultrasound image is formed by the back-scattered echoes from an approximately ellipsoidal region called the resolution cell\textsuperscript{8}. The interference of scatterers in a resolution cell creates the granular appearance of the ultrasound image, called speckle. Although of random appearance, speckle pattern is identical if the same object is scanned from the same direction and under the same focusing and frequency. Speckle characterization is essential in many areas of quantitative ultrasound. In this work, it is a prerequisite for speckle-based distance

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estimation. We use low order moments to discriminate fully developed speckle (FDS) patches versus coherent speckle patches.

\[ R = \frac{\langle A^v_r \rangle}{\sqrt{\langle A^{2v_r} \rangle - \langle A^{v_r} \rangle^2}} \]  
\[ S = \text{skewness} = \frac{\langle (A^v_r - \langle A^v_r \rangle)^3 \rangle}{\langle (A^{2v_r} - \langle A^{v_r} \rangle^2) \rangle^2} \]

where \( A \) is the amplitude of the ultrasound RF envelope in the analysis patch, \( v_r \) and \( v_s \) are the signal powers and \( \langle \cdots \rangle \) denotes the mean. Here we use \( v_r = 2v_s = 1 \). An elliptical discrimination function is calculated in the \( R-S \) plane by performing principal component analysis (PCA) on the data from simulated FDS patches. A patch is then classified as FDS if its \( R-S \) duple falls inside this ellipse.

Having found FDS patches in two ultrasound images, the correlation between them is used for estimating the distance between the two images. The \( R-S \) metric requires approximately 3500 pixels per patch (depending on the correlation of data), but such large patches (which are rectangles) of FDS are unlikely to be found in real tissue because of its inhomogeneity. Gee et al. proposed a heuristic technique that is robust to the lack of FDS patches in the ultrasound image. This method allows the calculation of the elevational distance for all patches of the image, regardless of their level of coherency, by measuring the axial and lateral correlation of each patch. Since the behavior of coherent reflectors in the elevational direction can be different from their behavior in the axial and lateral directions, the performance of the method can decline depending on the level of anisotropy of the tissue.

In, we proposed a fast algorithm to find irregularly shaped FDS patches and showed that this algorithm finds significantly more FDS patches. Here, we use beam steering as another technique to increase the number of FDS patches found in the image. This is achieved by obtaining more data from a certain region of tissue, hence reducing the size of the analysis patch. Having found such small FDS patches, we further use the steered images for better out-of-plane (elevational) motion estimation.

Coherent scattering causes the elevational distance measurement from the conventional correlation algorithms to be underestimated. Thus, distance measurement is limited to the patches of the ultrasound image that contain only FDS. To completely determine the out-of-plane degrees of freedom between two planes, at least three non-collinear pairs of such patches are required.

Since FDS patches are extremely rare in real tissue, these methods usually have a low accuracy and are only relevant in limited tissue types. Gee et al. proposed a heuristic technique that is robust to the lack of FDS patches in the ultrasound image. This method allows the calculation of the elevational distance for all patches of the image, regardless of their level of coherency, by measuring the axial and lateral correlation of each patch. Since the behavior of coherent reflectors in the elevational direction can be different from their behavior in the axial and lateral directions, the performance of the method can decline depending on the level of anisotropy of the tissue. The purpose of this work is to devise a method applicable to a various tissue types that accurately reconstructs 3D volumes from ultrasound images.

Recently, Laporte and Arbel have proposed probabilistic fusion of noisy out-of-plane motion estimation. This work is most similar to these works, in that it calculates the maximum likelihood estimate (MLE) of the out-of-plane motion. We also use beam steering to obtain more data and increase the accuracy of the out-of-plane motion estimation similar to.

2. METHODS

2.1. Combining Steered Images

We are looking for rectangular FDS patches using images acquired from the same location at different steering angles. The key idea is to combine data acquired from a certain region at different steering angles and therefore reducing the size of the analysis patch. Figure 2 shows two images acquired at 0 and \( \theta \) steering angles. A
Figure 1. (a) shows the three directions relative to the ultrasound probe. Out-of-plane direction and elevational direction are used interchangeably in this work. (b) shows acquisition of two ultrasound images at a distance of $\Delta z$. Ultrasound beam is in order of a millimeter wide. This wideness affects the resolution of ultrasound image in the lateral, $y$, and elevational, $z$, directions, as well as creating a granular pattern, called speckle. The size of the resolution cell in the axial direction, $x$, is determined by the wavelength of the ultrasound wave and is magnified in this image.

rectangle patch in the left image is warped into a parallelogram and is shifted in the steered right image. The position of the parallelogram can be simply found as a function of $\theta$, $x$ and $y$. Therefore, samples $n_X$ and $n_Y$ from the steered image correspond to samples $n_x$ and $n_y$ from the non-steered image and

$$
n_X = n_x - \frac{v_{US}}{2\nu} \cdot \frac{n}{w} \cdot \sin(\theta) \cdot n_y
$$

$$
n_Y = \frac{n_y}{\cos(\theta)}
$$

where $v_{US} = 1540000 \text{mm/s}$ is the speed of ultrasound in tissue, $\nu$ is the sampling frequency of the ultrasound machine, $n$ is the total number of the A-lines and $w$ is the width of image in mm. To find the correspondence of a patch, the correspondent of its four corners are found using these equations and applying nearest neighbor interpolation. The parallelogram connecting these four corners is the correspondent of the patch.

2.2. Maximum Likelihood Motion Estimation

Assume we have two parallel ultrasound images with ground truth out-of-plane distance $z$ (Figure 1), and that we have measured correlation coefficients $\rho_i$ for $i = 1 \cdots n$ patches between the two images (Figure 3). The goal is to find $\mu_z$ which is maximum likelihood estimate of $z$ given all the $\rho_i$ measurements. Let $\rho_i = f_i(z_i)$ be the calibration function that relates the out-of-plane motion $z_i$ to correlation coefficient $\rho_i$ for patch $i$ (each patch has a different calibration function depending on its depth, see Figure 6). Assuming that $\rho_i$ is drawn from a Gaussian distribution with mean $f(\mu_z)$ and variance $\sigma_i^2$, the conditional probability of $\rho_i$ is

$$
\Pr(\rho_i \mid \mu_z, \sigma_i^2) = \frac{1}{(2\pi \sigma_i^2)^{1/2}} e^{-\frac{(\rho_i - f_i(\mu_z))^2}{2\sigma_i^2}}
$$

(4)
Figure 2. Corresponding patches in images acquired with different steering angles. In the left, a patch is shown in the not-steered image. In the middle, the patch which corresponds to the same tissue is shown in the scan-converted steered image. In the right, the patch is shown in the raw steered image (not scan-converted).

Figure 3. The correlation coefficient $\rho_i$ is calculated between $n$ patches of the two images.

We assume that $\rho_i$ measurements are independent. Therefore, the conditional probability of observing all the $\rho_i$ values will be simply their multiplication. Looking at the product as a function of $\mu$ and $\sigma_i$ and taking its logarithm to convert multiplication to summation, we have the familiar log-likelihood equation

$$L(\vec{\rho} | \mu_z, \sigma^2) = -\sum_{i=1}^{n} \left[ \frac{1}{2} \log \sigma_i^2 + \frac{(\rho_i - f_i(\mu_z))^2}{\sigma_i^2} \right] + \frac{n}{2} \log(2\pi)$$

where $\vec{\rho}$ and $\vec{\sigma}^2$ are two vectors containing all the $\rho_i$ and $\sigma_i^2$ measurements. In the above equation, $\rho_i$ is the correlation of two corresponding patches and is known. $\sigma_i^2$ is also known: it is the variance of the correlation and is calculated in the calibration process (Figure 6). To find the ML estimate of the $\mu_z$, we differentiate this equation with respect to $\mu_z$ and set it to zero, arriving at

$$\sum_{i=1}^{n} f_i'(\mu_z)(\rho_i - f_i(\mu_z)) = 0$$

where $f'$ denotes the derivative of $f$. Unfortunately this equation is not easy to solve for $\mu_z$. Instead, let's transform $\rho_i$ to $z_i$ and write the log-likelihood functions in terms of $z_i$'s. Equation 5 becomes

$$L(\vec{z} | \mu_z, \sigma^2_z) = -\sum_{i=1}^{n} \left[ \frac{1}{2} \log \sigma_{zi}^2 + \frac{(z_i - \mu_z)^2}{\sigma_{zi}^2} \right] + \frac{n}{2} \log(2\pi)$$

where $\sigma_{zi}^2 = \frac{\sigma^2}{f'(\rho_i)^2}$ is the transformed variance in the $\rho$ domain ($\sigma_i$) to the $z$ domain ($\sigma_{zi}$). Differentiating with respect to $\mu_z$ and setting it to zero gives

$$\sum_{i=1}^{n} \frac{z_i - \mu_z}{\sigma_{zi}^2} = 0$$
which can be easily solved to give

\[ \mu_z = \frac{\sum_{i=1}^{n} \frac{z_i}{\sigma_i}}{\sum_{i=1}^{n} \frac{1}{\sigma_i^2}} \]

Finally, we utilize constraints in the images to enhance out-of-plane motion estimation. Many surgical procedures such as biopsy, drug delivery and brachytherapy involve inserting a needle into the tissue. The prior of needle continuity can be used to decrease the variance of the measured out of plane motion (we are assuming that the needle crosses US image plane and is not parallel to the image). Assume that the tip of the needle can be measured at each image with a variance of \( [\sigma_{\text{needle}}^*]^2 \), and that the angle of the needle with the normal of the ultrasound image (i.e the angle between the needle and the axes y in Figure 1) is \( \alpha \). Also, let \( \sigma_{\text{cor.}}^2 \) denote the variance of the out-of-plane motion estimation using the correlation method and \( \sigma_{\text{needle}} = \sigma_{\text{needle}}^*/\tan(\alpha) \). Assuming both noises are Gaussian, variance of the final estimate which combines the two estimates is \( \frac{\sigma_{\text{needle}}^2 \sigma_{\text{cor.}}^2}{\sigma_{\text{needle}}^2 + \sigma_{\text{cor.}}^2} \). It can be easily shown that this quantity is less than both \( \sigma_{\text{needle}}^2 \) and \( \sigma_{\text{cor.}}^2 \), meaning that the resulting variance is less than both initial variances.

2.3. Calibration and Data Acquisition

The system operates in two distinct modes - calibration mode and image-based 3DUS reconstruction mode (Figure 4). Both will be described from a process flow perspective. In the calibration mode, information necessary to calibrate the distance estimations is collected (Figure 4). To this end, the robot control component steps the robot through a series of precisely defined positions and triggers the acquisition of a single US frame (RF data) at each position from a homogeneous fully developed speckle (FDS) phantom. These frames are associated with their respective coordinates and stored for offline use. Then, the software system reads the batch of frames and positions and subdivides the frames into distinct subpatches. Pairs of patches from the same location originating from frames at different distances are correlated, thus creating a set of (strictly monotonous) calibration (or decorrelation) curves \( x, y(d) \). These curves depend on the characteristics of the selected probe, the imaging frequency, and the image location \( x, y \) (in particular the depth \( y \)) of the respective patches. Currently, the offline calibration process takes 2-3 minutes including scan time to generate the needed calibration curves (decorrelation curves). Before this recent development, manual data collection and offline processing using MATLAB scripts used to take many hours of effort.
3. EXPERIMENTAL SETUP

The ultrasound RF data was sampled with a robot-based system in order to achieve reliable, high-accuracy ground truth readings for the displacements. This will give us the images we need for calibration and also the gives us the ground truth when we reconstruct the volume. This process yielded a series of planar-parallel RF slices through the respective phantom, in a fashion somewhat comparable to a freehand sweep. The phantoms were positioned within the workspace of a high-precision three degrees-of-freedom (DoF) cartesian robot stage (DMC-21x3 with three servo motor stages, by Galil Motion Control; relative accuracy better 0.005 mm). For calibration, the stage translated the probe to new positions every $\Delta x = 0.05$ mm apart, then triggered RF slice acquisition via a TTL signal connected to the ultrasound machine’s ECG trigger port, where the data was written to file. For calibration data acquisition, a FDS phantom is imaged. For volume reconstruction, real tissue (beef steak) is used. Figure 5 shows the experimental setup.

An Ultrasonix ultrasound machine (Burnaby, BC) with a sampling frequency of $\nu = 20$MHz is used to acquire RF data. To calibrate the rate of image decorrelation with out-of-plane motion, RF data of 5x80 parallel frames were acquired from a FDS phantom at an elevational distance of 0.05 mm between consecutive images: five frames at each location with steering angles of -5, -2.5, 0, 2.5 and 5.5 degrees. The experimental setup is shown in Figure 5: the probe is moved with a micrometer with the accuracy of .005 mm. Calibration results showed that the decorrelation rate is not affected by beam steering.

Out-of-plane motion estimation was performed on ex-vivo beef steak tissue. 4x80 RF frames at an elevational distance of 0.05 mm between consecutive frames were acquired using the setup shown in Figure 5: four images at each location with $-5^\circ$, $-2.5^\circ$, $0^\circ$, $2.5^\circ$ and $5^\circ$ steering angles.
4. OUT-OF-PLANE MOTION ESTIMATION

The correlations are calculated using Pearson’s linear correlation coefficient $\rho$

$$\rho(W, Z) = \frac{\Sigma w_i z_i - N \mu_w \mu_z}{\sqrt{(\Sigma w_i^2 - N \mu_w^2) (\Sigma z_i^2 - N \mu_z^2)}}$$ (10)

where $w_i$ and $z_i$, $i = 1 \cdots N$, are the intensity values of each pixel in patches $W$ and $Z$, $N$ is the total number of pixels and $\mu_w$ and $\mu_z$ are the means of the intensity values of patches $W$ and $Z$ respectively.

Patches that are closest to being FDS are selected as described in. Figure 8 shows the results of reconstructing out-of-plane motion using the correlation values. (a) and (b) are obtained by combining the two images with $\pm 2.5^\circ$ steering angle at each location, while (c) and (d) are obtained by combining the two images with $\pm 5^\circ$ steering angle at each location. The results show that using the MLE method slightly reduces both the underestimation error and the variance of the out-of-plane measurements. In addition, it can be seen from (b) and (d) that the needle constraint reduces the variance in the measurements.

5. DISCUSSION AND CONCLUSION

Out-of-plane motion estimation is only studied here for a fixed distance between two frames, 0.4mm. A study of accuracy as the distance varies gives insight for optimum frame selection. In freehand experiments the images are not parallel as they are in our experiments, and therefore the rotations between the images need to be found. We showed before that performing beam steering significantly increases the accuracy of out-of-plane motion estimation. In this work, we showed that MLE can also be used to enhance the out-of-plane motion estimation.
Figure 8. relative error and standard deviation of the sensorless measurements. (a) The relative error. Reconstruction is performed using two steered images at ±2.5°. (b) The standard deviation of the measurements. W/O MLE refers to without MLE, W/O constraint refers to without utilizing the needle continuity constraint, and W MLE refers to with MLE. (c) and (d) are corresponding errors and variances with two steered images at ±5°.
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