

Mixed Stochastic and Event Flows

Brownian Motion Modeling for Simulation Dynamics

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Scalable Network Simulation Models

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- Future network design and modeling requires large scale, high fidelity simulations capability.
- Training requires real-time speedup of network simulations.
- Parallelization of network simulations not always useful due to lack of topological communities of interest.
- Hybrid analytic/event simulations appear to be an attractive alternative.

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- Network models:
 - Partitioned models, e.g., packet edge and analytic core
 - **Mixed node models, i.e., packet and analytic traffic mixed at each network queue.**
- Analytic models:
 - Deterministic models, e.g., dynamics described by deterministic differential equations.
 - **Stochastic models, e.g., Brownian motion models of queue dynamics.**

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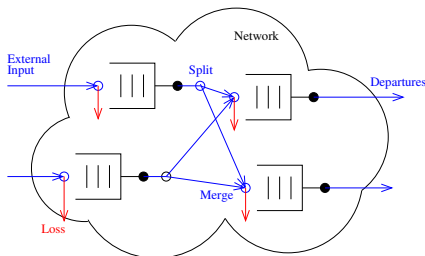
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Simulation of network of queues.

- Mixing the two fundamentally different traffic types at a single, finite queue.
- Time dependent models of finite-sized queue dynamics.
- Splitting and merging mixed-traffic flows within network simulation.

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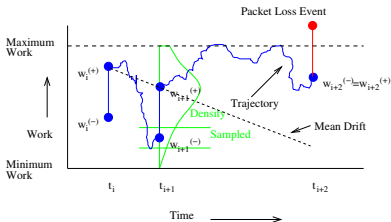
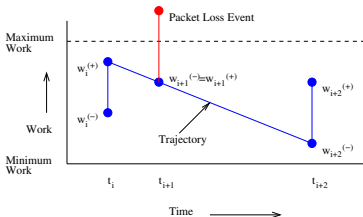
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Contrasting deterministic versus stochastic fluid mixing at queue.

- Cumulative Distribution Function for a $GI/G/1/K$ queueing system, $F(w, t|w_0, t = 0)$ satisfies the Fokker-Planck Equation [Kobayashi, 1974a], [Kobayashi, 1974b] and [Heyman, 1975]

$$\frac{\partial}{\partial t} F = -m \frac{\partial F}{\partial w} + \frac{1}{2} \sigma^2 \frac{\partial^2 F}{\partial w^2} \quad (1)$$

where $m = \lambda_s - \mu$, and $\sigma^2 = \lambda_s \times C_v^2 + \mu \times C_{v,\mu}^2$.

- The Fokker-Planck equation has an analytic solution

$$\begin{aligned} F(w, t|w_0, t = 0) &= \alpha \times \Phi\left(\frac{w - w_0 - mt}{\sigma\sqrt{t}}\right) \\ &+ \beta \times e^{2mw/\sigma^2} \Phi\left(\frac{-w - w_0 - mt}{\sigma\sqrt{t}}\right) \end{aligned} \quad (2)$$

$$\Phi\left(\frac{\pm w - w_0 - mt}{\sigma\sqrt{t}}\right) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\pm w - w_0 - mt}{\sigma\sqrt{t}}} e^{-x^2/2} dx \quad (3)$$

- Initial Conditions:

$F(w, t = 0) = H(w - w_0)$; $H(x)$ is the Heavy-side Function.

- Upper and lower limits on work in system:

Common BCs -

$$\lim_{w \rightarrow 0} F(w, t | w_0, t = 0) = 0 \quad (4)$$

$$\lim_{w \rightarrow w_{max}} F(w, t | w_0, t = 0) = 1 \quad (5)$$

Our alternative BCs -

$$\lim_{w \rightarrow \max[0, w_0 - \mu t]} F(w, t | w_0, t = 0) = 0 \quad (6)$$

$$\lim_{w \rightarrow w_{max}} F(w, t | w_0, t = 0) = 1 \quad (7)$$

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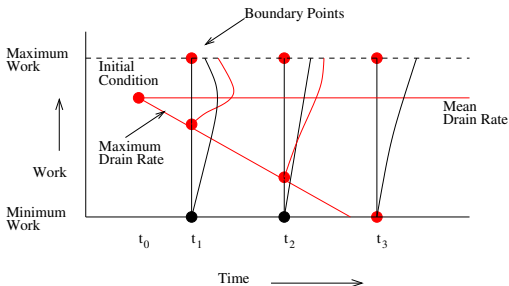
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Alternative BCs and their impact of density functions versus time.

- Buffer size sets maximum and minimum limits to work in queue.
- Maximum drain rate (assuming no arrivals) sets short term time-dependent limit on the minimum work in queue.

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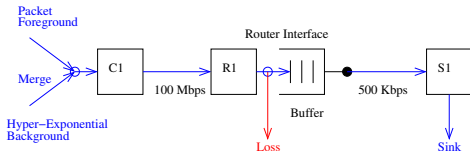
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Simple simulation model in GTNets.

- Background is hyper-exponential arrival with deterministic service, modeling UDP packets
- Foreground is exponential arrival with deterministic service, modeling UDP packets
- Foreground is later modeled as TCP stream
- Investigate foreground packet delay (UDP and TCP), loss (UDP and TCP), and goodput (TCP)

UDP Delay and Loss Results

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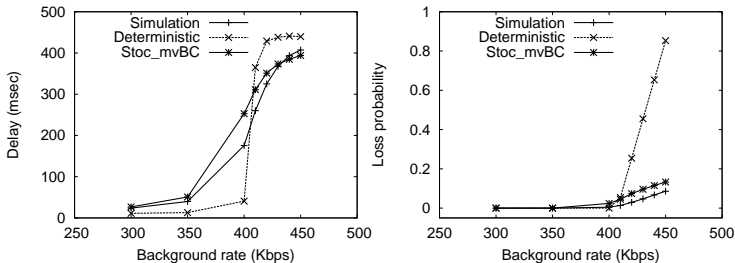
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UDP delay and loss versus rate in mixed traffic-type queue.

- Deterministic fluid model has no mechanism to allow foreground traffic buffer access at high utilization.
- Stochastic model allows foreground traffic access even in overload situations.

UDP Delay Results versus C_v

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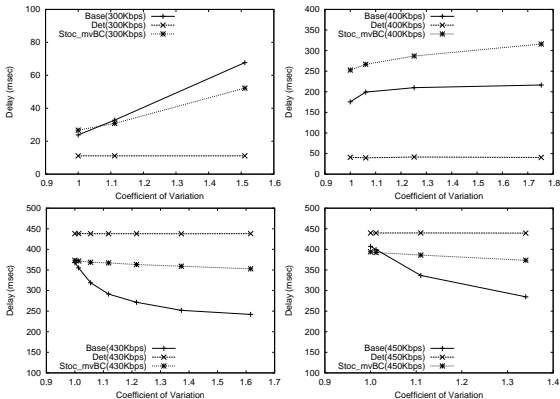
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UDP delay versus C_v in mixed traffic-type queue.

TCP Goodput Results

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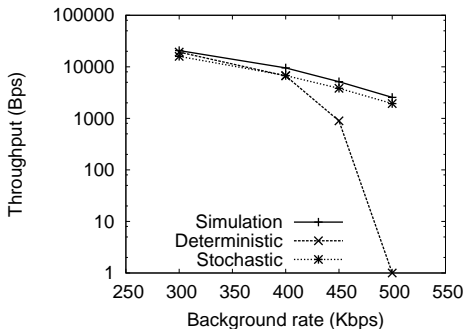
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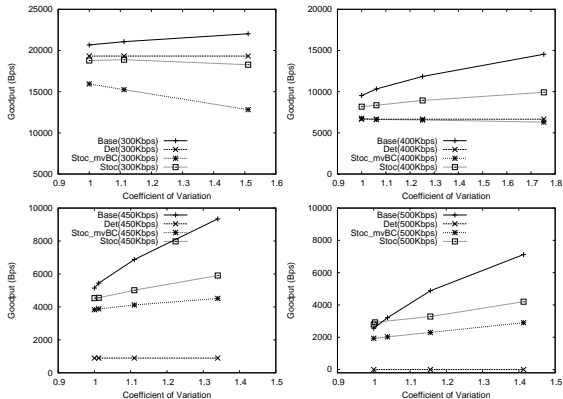
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TCP goodput versus rate in mixed traffic-type queue.

- Stochastic model matches well with simulation results for TCP dynamics.



TCP goodput versus C_v in mixed traffic-type queue.

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- Improve Mixed Queue Models:
 - Not happy with $C_v \neq 1$ results.
 Possible solution: discretize distribution [Kobayashi, 1974b] or investigate scaling laws for diffusional drift and variance.
 - Develop the background loss models.
 Possible approach is based upon applications of Bayes Theorem (see below).
- Develop Network Flow Models:
 - Only investigated mixing at single node models to date.
 - Leverage literature of network queueing, e.g., [Whitt, 1995], [Kobayashi and Mark, 2002], others.

Loss Models of Fluid

Application of Bayes Theorem

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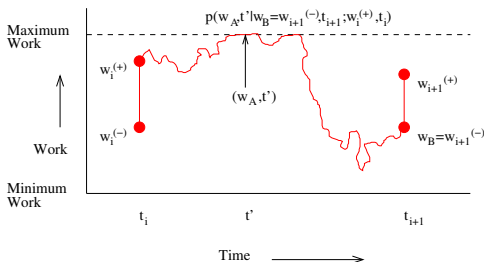
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- Fluid loss ($t \rightarrow t'$) = $\lambda \int_t^{t'} \rho(w_{max}, x) dx$

- Bayes Equation: $P(A|B) = P(B|A)P(A)/P(B)$

$$\rho(w_A, t' | w_B = w_{i+1}^{(-)}, t_{i+1}; w_i^{(+)}, t_i) = \frac{\rho(w_B = w_{i+1}^{(-)}, t_{i+1} | w_A, t') \rho(w_A, t' | w_i^{(+)}, t_i)}{\rho(w_B = w_{i+1}^{(-)}, t_{i+1} | w_i^{(+)}, t_i)} \quad (8)$$

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- Initial investigations into mixed, hybrid stochastic simulation models
- Much work to be done:
 - Improve $C_v \neq 1$ results
 - Develop time-dependent fluid loss models
 - Develop network flow models, i.e. time-dependent network calculus
- Outlook
 - Initial simulation results are encouraging
 - Need much more development and simulation studies results of time-dependent dynamics

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[Cole, R.G., Riley, G., Cansever, D. and W. Yurcik,](#)

Stochastic Process Models for Packet/Analytic-Based Network Simulations, IEEE/ACM Principles of Distributed Simulation (PADS), Rome, Italy, June 2008.



[Heyman, D.,](#)

A Diffusion Model Approximation for the GI/G/1 Queue in Heavy Traffic, Bell Labs Technical Journal, Vol. 54, No. 9, November 1975.



[Kobayashi, H.,](#)

Applications of the Diffusion Approximation to Queueing Networks, I. Equilibrium Queue Distributions, Journal of the Association for Computing Machinery, Vol. 21, No. 2, 1974.



[Kobayashi, H.,](#)

Applications of the Diffusion Approximation to Queueing Networks, I. Nonequilibrium Distributions and Computer Modeling, Journal of the Association for Computing Machinery, Vol. 21, No. 3, 1974.



[H. Kobayashi and B. L. Mark,](#)

A Unified Theory for Queuing and Loss-Queueing Networks, IFORS'99 Conference, Beijing, August 16-20, 1999.



[H. Kobayashi and B. L. Mark,](#)

Generalized Loss Models and Queueing-Loss Networks, Int. Trans. in Operational Research, Vol. 9, No. 1, pp. 97-112, January 2002.



[Whitt, W.,](#)

Variability Functions for Parametric-Decomposition Approximations of Queueing Networks. Management Science, vol. 41, No. 10, pp. 1704-1715, 1995.