Game Playing

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(slides by Philipp Koehn)

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Outline

• Games

• Perfect play
  – minimax decisions
  – $\alpha-\beta$ pruning

• Resource limits and approximate evaluation

• Games of chance

• Games of imperfect information
games
Games vs. Search Problems

• “Unpredictable” opponent ⇒ solution is a strategy specifying a move for every possible opponent reply

• Time limits ⇒ unlikely to find goal, must approximate

• Plan of attack:
  – computer considers possible lines of play (Babbage, 1846)
  – algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
  – finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
  – first Chess program (Turing, 1951)
  – machine learning to improve evaluation accuracy (Samuel, 1952–57)
  – pruning to allow deeper search (McCarthy, 1956)
## Types of Games

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Game Tree (2-player, Deterministic, Turns)

MAX (X)

MIN (O)

MAX (X)

MIN (O)

TERMINAL

Utility

-1 0 +1
Simple Game Tree

- 2 player game
- Each player has one move
- You move first
- Goal: optimize your payoff (utility)
minimax
Minimax

- Perfect play for deterministic, perfect-information games

- Idea: choose move to position with highest minimax value
  = best achievable payoff against best play

- E.g., 2-player game, one move each:
Minimax Algorithm

function MINIMAX-DECISION(state) returns an action
    inputs: state, current state in game
    return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))

function MAX-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    \[ v \leftarrow -\infty \]
    for a, s in SUCCESSORS(state) do \[ v \leftarrow \max(v, \text{MIN-VALUE}(s)) \]
    return v

function MIN-VALUE(state) returns a utility value
    if TERMINAL-TEST(state) then return UTILITY(state)
    \[ v \leftarrow \infty \]
    for a, s in SUCCESSORS(state) do \[ v \leftarrow \min(v, \text{MAX-VALUE}(s)) \]
    return v
Properties of Minimax

- **Complete?** Yes, if tree is finite

- **Optimal?** Yes, against an optimal opponent. Otherwise??

- **Time complexity?** $O(b^m)$

- **Space complexity?** $O(bm)$ (depth-first exploration)

- For Chess, $b \approx 35$, $m \approx 100$ for “reasonable” games
  $\Rightarrow$ exact solution completely infeasible

- But do we need to explore every path?
$\alpha-\beta$ Pruning Example

[Diagram of a pruned game tree with values 3, 12, and 8 at different nodes.]
$\alpha-\beta$ Pruning Example

MIN

3

3

12

8
\(\alpha - \beta\) Pruning Example

MAX

MIN

3
12
8

\(\geq 3\)
\( \alpha-\beta \) Pruning Example

```
\begin{center}
\begin{tikzpicture}
  \node (root) at (0,0) {$\geq 3$};
  \node (min1) at (-2,-2) {$\leq 2$};
  \node (max1) at (-4,-4) {$3$};
  \node (max2) at (-2,-4) {$12$};
  \node (max3) at (-1,-4) {$8$};
  \node (max4) at (0,-4) {$2$};

  \draw (root) -- (min1);
  \draw (min1) -- (max1);
  \draw (min1) -- (max2);
  \draw (min1) -- (max3);
  \draw (min1) -- (max4);

  \node at (-3,-5.5) {MIN};
  \node at (-5,-5.5) {MAX};

\end{tikzpicture}
\end{center}
```
\( \alpha-\beta \) Pruning Example
$\alpha$$\beta$ Pruning Example

![Game tree diagram showing an \(\alpha\beta\) pruning example.](attachment:game_tree_diagram.png)
$\alpha-\beta$ Pruning Example

The diagram illustrates a game tree with MAX and MIN nodes, where MAX tries to maximize the value, and MIN tries to minimize it. The pruning example shows how $\alpha-\beta$ pruning eliminates branches from the tree that cannot lead to a better outcome for either player.
Why is it Called $\alpha$–$\beta$?

- $\alpha$ is the best value (to MAX) found so far off the current path
- If $V$ is worse than $\alpha$, MAX will avoid it $\Rightarrow$ prune that branch
- Define $\beta$ similarly for MIN
The $\alpha-\beta$ Algorithm

\begin{verbatim}
function AMPLA-BETA-DECISION(state) returns an action
    return the $a$ in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))

function MAX-VALUE(state, $\alpha$, $\beta$) returns a utility value
    inputs: state, current state in game
            $\alpha$, the value of the best alternative for MAX along the path to state
            $\beta$, the value of the best alternative for MIN along the path to state
    if TERMINAL-TEST(state) then return UTILITY(state)
    $v \leftarrow -\infty$
    for $a$, $s$ in SUCCESSORS(state) do
        $v \leftarrow \text{MAX}(v, \text{MIN-VALUE}(s, s, \alpha, \beta))$
        if $v \geq \beta$ then return $v$
        $\alpha \leftarrow \text{MAX}(\alpha, v)$
    return $v$

function MIN-VALUE(state, $\alpha$, $\beta$) returns a utility value
    same as MAX-VALUE but with roles of $\alpha$, $\beta$ reversed
\end{verbatim}
Properties of $\alpha-\beta$

- Safe: Pruning **does not** affect final result
- Good move ordering improves effectiveness of pruning
- With “perfect ordering,” time complexity $= O(b^{m/2})$
  \[ \Rightarrow \textbf{doubles} \text{ solvable depth} \]
- A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)
- Unfortunately, $35^{50}$ is still impossible!
Solved Games

• A game is solved if optimal strategy can be computed

• Tic Tac Toe can be trivially solved

• Biggest solved game: Checkers
  – proof by Schaeffer in 2007
  – both players can force at least a draw

• Most games (Chess, Go, etc.) too complex to be solved
resource limits
Resource Limits

- Standard approach:
  - Use `CUTOFF-TEST` instead of `TERMINAL-TEST`
    e.g., depth limit (perhaps add quiescence search)
  - Use `EVAL` instead of `UTILITY`
    i.e., evaluation function that estimates desirability of position

- Suppose we have 100 seconds, explore $10^4$ nodes/second
  $\Rightarrow$ $10^6$ nodes per move $\approx 35^{8/2}$
  $\Rightarrow$ $\alpha-\beta$ reaches depth 8 $\Rightarrow$ pretty good Chess program
• For Chess, typically linear weighted sum of features

\[ \text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s) \]

e.g., \( f_1(s) = \text{(number of white queens) – (number of black queens)} \)
Evaluation Function for Chess

- Long experience of playing Chess

⇒ Evaluation of positions included in Chess strategy books
  - bishop is worth 3 pawns
  - knight is worth 3 pawns
  - rook is worth 5 pawns
  - good pawn position is worth 0.5 pawns
  - king safety is worth 0.5 pawns
  - etc.

- Pawn count → weight for features
Learning Evaluation Functions

- Designing good evaluation functions requires a lot of expertise

- Machine learning approach
  - collect a large database of games play
  - note for each game who won
  - try to predict game outcome from features of position
  ⇒ learned weights

- May also learn evaluation functions from self-play
Some Concerns

- **Quiescence**
  - position evaluation not reliable if board is unstable
  - e.g., Chess: queen will be lost in next move
  - deeper search of game-changing moves required

- **Horizon Effect**
  - adverse move can be delayed, but not avoided
  - search may prefer to delay, even if costly
Forward Pruning

- Idea: avoid computation on clearly bad moves
- Cut off searches with bad positions before they reach max-depth
- Risky: initially inferior positions may lead to better positions
- Beam search: explore fixed number of promising moves deeper
Lookup instead of Search

- Library of opening moves
  - even expert Chess players use standard opening moves
  - these can be memorized and followed until divergence

- End game
  - if only few pieces left, optimal final moves may be computed
  - Chess end game with 6 pieces left solved in 2006
  - can be used instead of evaluation function
Digression: Exact Values do not Matter

- Behaviour is preserved under any *monotonic* transformation of $EVAL$

- Only the order matters:
  payoff in deterministic games acts as an *ordinal utility* function
Deterministic Games in Practice

- **Checkers**: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions. Weakly solved in 2007 by Schaeffer (guaranteed draw).

- **Chess**: Deep Blue defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

- **Go**: In 2016, computer using a neural network for the board evaluation function, was able to beat the human Go champion for the first time. Given the huge branching factor ($b > 300$), Go was long considered too difficult for machines.
games of chance
Nondeterministic Games: Backgammon
Nondeterministic Games in General

- In nondeterministic games, chance introduced by dice, card-shuffling
- Simplified example with coin-flipping:
Algorithm for Nondeterministic Games

- `EXPECTIMINIMAX` gives perfect play

- Just like `MINIMAX`, except we must also handle chance nodes:

  ...  
  if `state` is a `MAX` node then
    return the highest `EXPECTIMINIMAX-VALUE` of `SUCCESSORS(state)`
  if `state` is a `MIN` node then
    return the lowest `EXPECTIMINIMAX-VALUE` of `SUCCESSORS(state)`
  if `state` is a chance node then
    return average of `EXPECTIMINIMAX-VALUE` of `SUCCESSORS(state)`
  ...

...
Pruning in Nondeterministic Game Trees

A version of $\alpha-\beta$ pruning is possible:
Pruning in Nondeterministic Game Trees

A version of $\alpha$-$\beta$ pruning is possible:
Pruning in Nondeterministic Game Trees

A version of \( \alpha\beta \) pruning is possible:
Pruning in Nondeterministic Game Trees

A version of $\alpha$-$\beta$ pruning is possible:
Pruning in Nondeterministic Game Trees

A version of $\alpha$-$\beta$ pruning is possible:
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A version of $\alpha$-$\beta$ pruning is possible:
Pruning in Nondeterministic Game Trees

Terminate, since right path will be worth on average.
Pruning with Bounds

More pruning occurs if we can bound the leaf values (0,1,2)
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Nondeterministic Games in Practice

- Dice rolls increase $b$: 21 possible rolls with 2 dice
- Backgammon $\approx$ 20 legal moves (can be 6,000 with 1-1 roll)

\[
\text{depth 4} = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9
\]

- As depth increases, probability of reaching a given node shrinks
  $\Rightarrow$ value of lookahead is diminished
- $\alpha$–$\beta$ pruning is much less effective
- TDGAMMON uses depth-2 search + very good EVAL
  $\approx$ world-champion level
Digression: Exact Values Do Matter

- Behaviour is preserved only by positive linear transformation of $E_{\text{VAL}}$

- Hence $E_{\text{VAL}}$ should be proportional to the expected payoff
imperfect information
Games of Imperfect Information

• E.g., card games, where opponent’s initial cards are unknown

• Typically we can calculate a probability for each possible deal

• Seems just like having one big dice roll at the beginning of the game

• Idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals

• Special case: if an action is optimal for all deals, it’s optimal.
Commonsense Counter-Example

- Road A leads to a small heap of gold pieces ($)
  Road B leads to a fork:
    take the left fork and you’ll find a mound of jewels ($$$);
    take the right fork and you’ll be run over by a bus.

- Road A leads to a small heap of gold pieces ($)
  Road B leads to a fork:
    take the left fork and you’ll be run over by a bus;
    take the right fork and you’ll find a mound of jewels ($$$);

  ⇒ does not matter if jewels are left or right on road B, it’s always better choice

- Road A leads to a small heap of gold pieces ($);
  Road B leads to a fork:
    guess correctly and you’ll find a mound of jewels ($$$);
    guess incorrectly and you’ll be run over by a bus.

  ⇒ it does matter if we know where forks on road B lead to
Proper Analysis

• Intuition that the value of an action is the average of its values in all actual states is **WRONG**

• With partial observability, value of an action depends on the **information state or belief state** the agent is in

• Can generate and search a tree of information states

• Leads to rational behaviors such as
  – acting to obtain information
  – signalling to one’s partner
  – acting randomly to minimize information disclosure
Computer Poker

• Hard game
  – imperfect information — including bluffing and trapping
  – stochastic outcomes — cards drawn at random
  – partially observable — may never see other players hand when they fold
  – non-cooperative multi-player — possibility for coalitions

• Few moves (fold, call, raise), but large number of stochastic states

• Relative balance of deception plays very important
  also: when to bluff

⇒ There is no single best move

• Need to model other players (style, collusion, patterns)

• Hard to evaluate (not just win/loss, different types of opponents)
Summary

- Games are fun to work on

- They illustrate several important points about AI
  - perfection is unattainable ⇒ must approximate
  - good idea to think about what to think about
  - uncertainty constrains the assignment of values to states
  - optimal decisions depend on information state, not real state

- Games are to AI as grand prix racing is to automobile design