Introduction to Computational Geometry

Michael Kazhdan

(600.459)
What are we studying?

• Algorithms and data-structures for solving geometric problems.

• Following O’Rourke, “Computational Geometry in C (2nd Edition)”:  
  http://cs.smith.edu/~orourke/books/compgeom.html  
  http://cs.smith.edu/~orourke/books/ftp.html
More Specifically

- Polygon Triangulation
- Polygon Partitioning
- Convex Hulls in 2D/3D
- Voronoi Diagrams
- Arrangements
- Search & Intersection
- Motion Planning
Additional Resources

QHull: www.qhull.org/

ANN: www.cs.umd.edu/~mount/ANN/

Triangle: www.cs.cmu.edu/~quake/triangle.html

TetGen: wias-berlin.de/software/tetgen/

CGAL: www.cgal.org/
Syllabus

When: MW @ 1:30 – 2:45
Where: Hodson 316
Web: http://www.cs.jhu.edu/~misha/Spring20/
TA: Tommy Mitchell
Assignments: Yes
Exams: No
Example: Convex Hulls in 2D

Given: A set $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$.

Output: An ordered subset $CH = \{p_{i_1}, \ldots, p_{i_m}\} \subset P$ describing the convex hull.

$P = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$

$CH = \{p_1, p_2, p_7, p_6, p_3\}$
Example: Convex Hulls in 2D

Note:

Hull edges have the property that all other points are to the left of the edge.

\[ CH = \{ p_1, p_2, p_7, p_6, p_3 \} \]
Example: Convex Hulls in 2D

Take 1:

1. \( E \leftarrow \emptyset \)
2. \( \forall (p, q) \in P \times P \text{ with } p \neq q \)
3. \( \text{hullEdge} \leftarrow \text{true} \)
4. \( \forall r \in P \text{ with } r \neq p \text{ and } r \neq q \)
5. \( \text{If } r \text{ is right of } \overrightarrow{pq}: \text{hullEdge} \leftarrow \text{false} \)
6. \( \text{If } \text{hullEdge}: E \leftarrow E \cup \overrightarrow{pq} \)
7. Construct \( CH \) from \( E \).
Example: Convex Hulls in 2D

Take 1:

1. $E \leftarrow \emptyset$
2. $\forall (p, q) \in P \times P \text{ with } p \neq q$
3. $hullEdge \leftarrow true$
4. $\forall r \in P \text{ with } r \neq p \text{ and } r \neq q$
5. If $r$ is right of $\overline{pq}$: $hullEdge \leftarrow false$
6. If $hullEdge$: $E \leftarrow E \cup \overline{pq}$
7. Construct $CH$ from $E$. 

Correctness?
Robustness?
Efficiency?
Example: Convex Hulls in 2D

Note:

Successive edges on the hull make left turns.

\[ CH = \{ p_1, p_2, p_7, p_6, p_3 \} \]
Example: Convex Hulls in 2D

**Take 2: (Graham Scan)**

1. Find bottom-most (left-most) point $p$
2. Sort remaining points in $P$ by slope
3. $CH \leftarrow \{p, p_1\}$
4. For $i \in \{2, ..., n - 1\}$
5. $CH \leftarrow CH \cup \{p_i\}$
6. While last three points don’t make a left turn
7. Delete the middle of the last three points

Correctness? Robustness? Efficiency?
For Next Class (Wednesday)

Chapter 1: Polygon Triangulation