FFTs in Graphics and Vision

Conclusion

Announcements

Assignment 3 due May 9th.

Key Ideas

Functions Are Vectors

We can think of functions as (complex) vectors in a high-dimensional space.

Representations

Translations and rotations of functions can be thought of as linear, norm-preserving, transformations.

Irreducible Representations

To understand how translations and rotations act on functions, we strive to decompose the space of functions into the smallest sub-spaces in which these transformations are contained.

Essential Facts

Schur's Lemma (Corollary)

If the group is commutative, the irreducible representations are all one-dimensional.

Homogenous Polynomials

Homogenous polynomials of fixed degree are sub-representations.

Self-Adjoint Operators

If a linear operator is self-adjoint (symmetric) there is an orthogonal basis of eigenvectors.

Commuting Self-Adjoint Operators

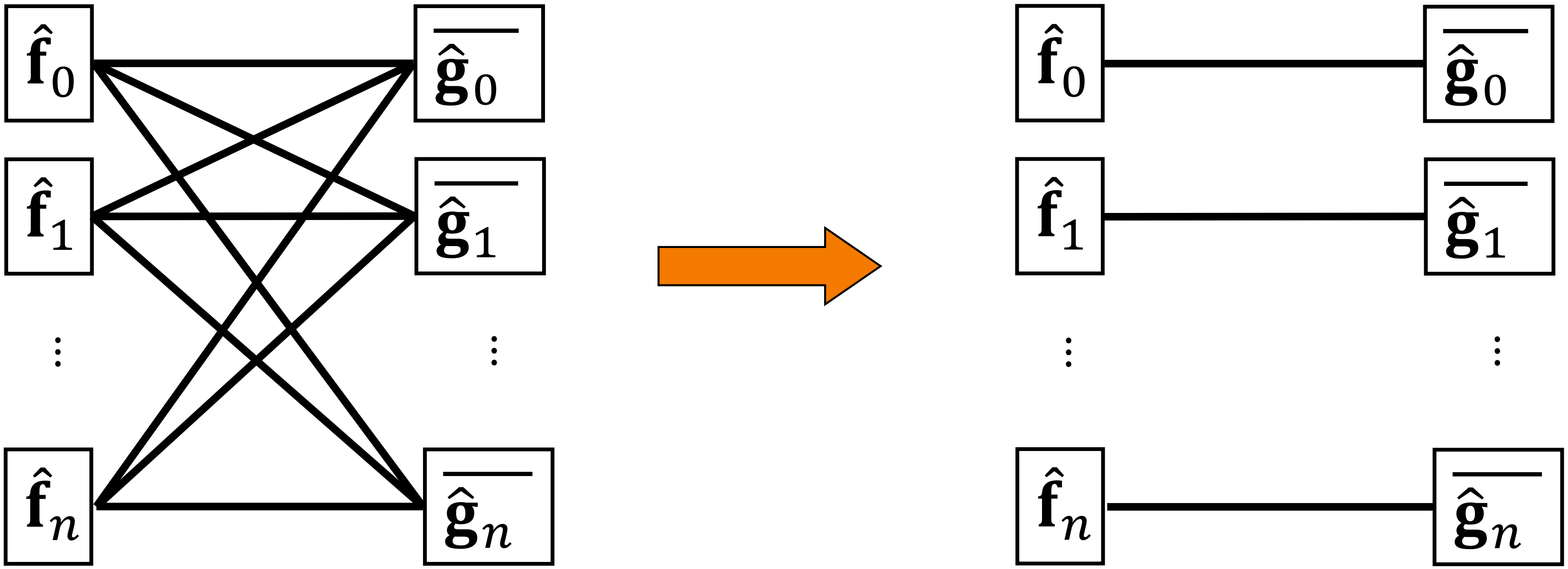
The spaces of eigenvectors of the operator with the same eigenvalue are sub-representation.

Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency:

$$D_{f,g}(\alpha) = \sum_k \hat{\mathbf{f}}_k \cdot \overline{\hat{\mathbf{g}}_k} \cdot D_k(\alpha)$$



Implications / Applications

<u>Fast Correlation</u>:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency.

This reduces the implementation of correlation to:

1. Computing a forward frequency transform
2. Doing the intra-frequency cross multiplication
3. Computing an inverse frequency transform

Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency.

For Translational Correlation:

1. Computing the FFT: $O(N \log N)$
2. Doing the multiplication: $O(N)$
3. Computing the inverse FFT: $O(N \log N)$

Brute force would have been $O(N^2)$.

Optimal would be $O(N)$.

Implications / Applications

Fast Correlation:

For correlation, we only need to perform cross-multiplication of coefficients in the same frequency.

For Rotational Correlation:

1. Computing the SHT: $O(N^2 \log^2 N)$
2. Doing the multiplication: $O(N^3)$
3. Computing the inverse WDT: $O(N^3 \log^2 N)$

Brute force would have been $O(N^5)$.

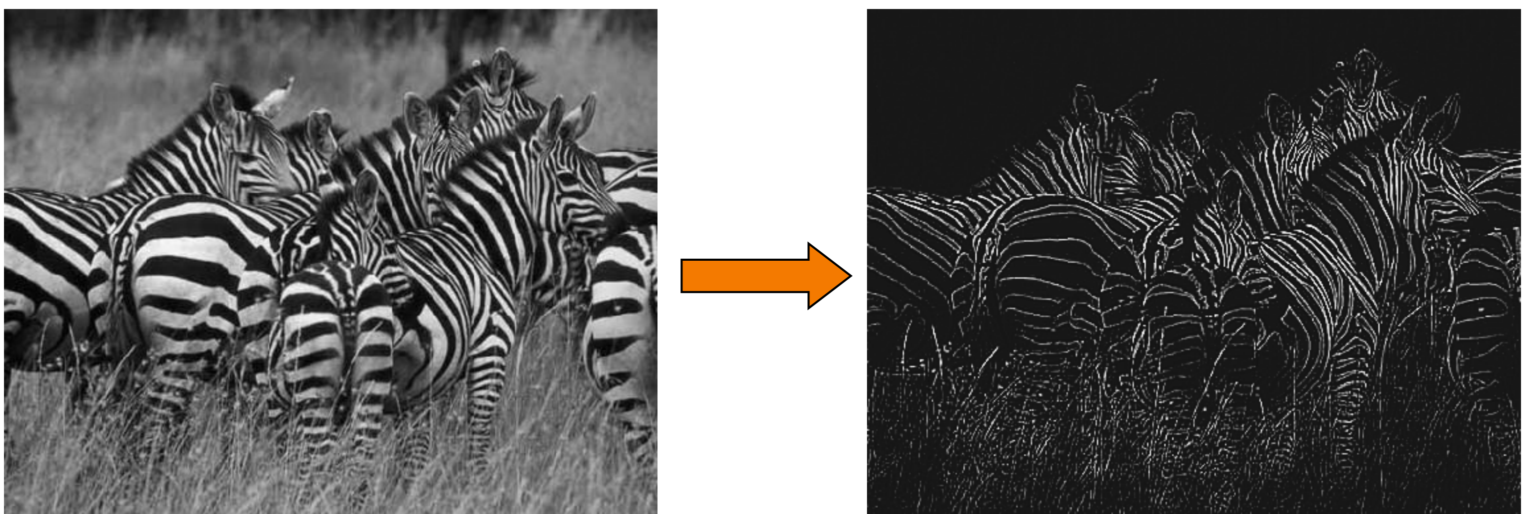
Optimal would have been $O(N^3)$.

Implications / Applications

<u>Fast Correlation (Applications)</u>:

For image processing, we have seen applications of correlation in:

- Filtering



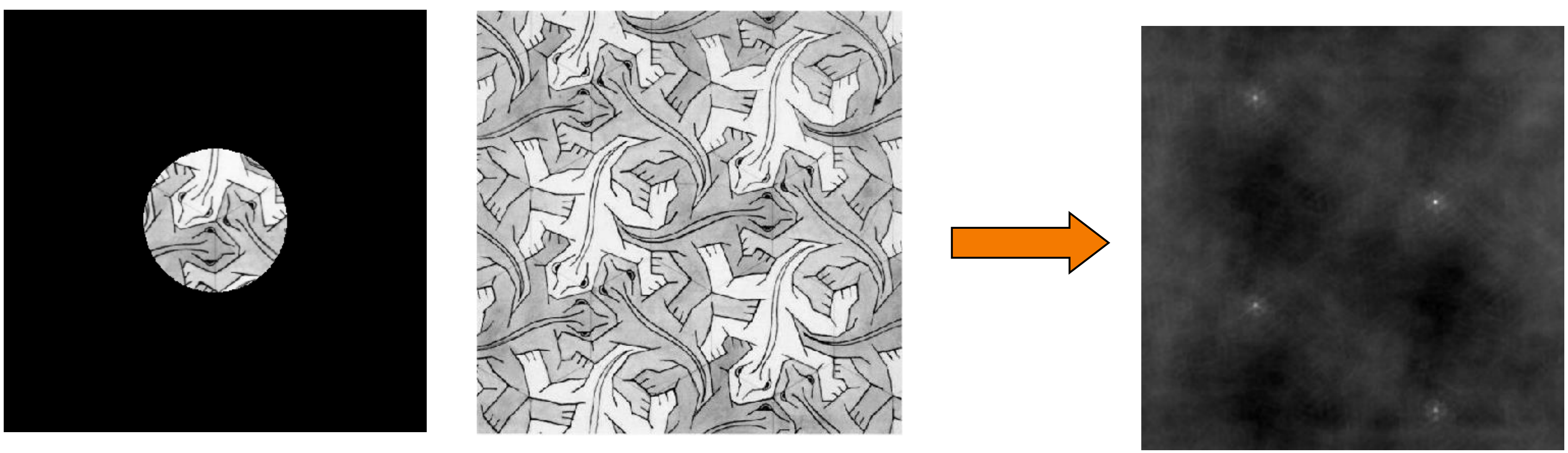
Edge Detection

Implications / Applications

<u>Fast Correlation (Applications)</u>:

For image processing, we have seen applications of correlation in:

- Filtering
- Pattern Recognition



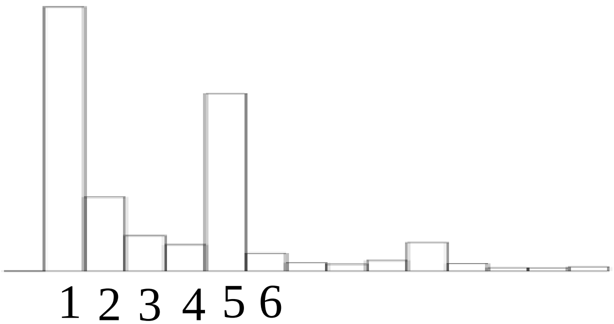
Template Matching

Implications / Applications

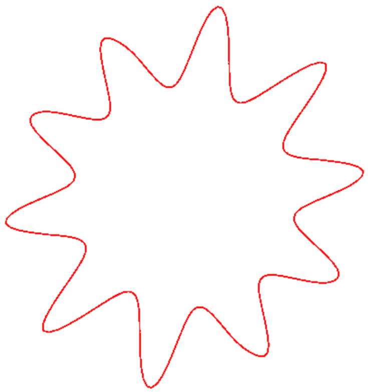
<u>Fast Correlation (Applications)</u>:

For image processing, we have seen applications of correlation in:

- Filtering
- Pattern Recognition
- Symmetry Detection



Rotational Symmetry



Reflective Symmetry

Implications / Applications

Fast Correlation (Applications):

We have even seen some unexpected applications resulting from the fact that the Laplacian is a symmetric operator commuting with translation in:

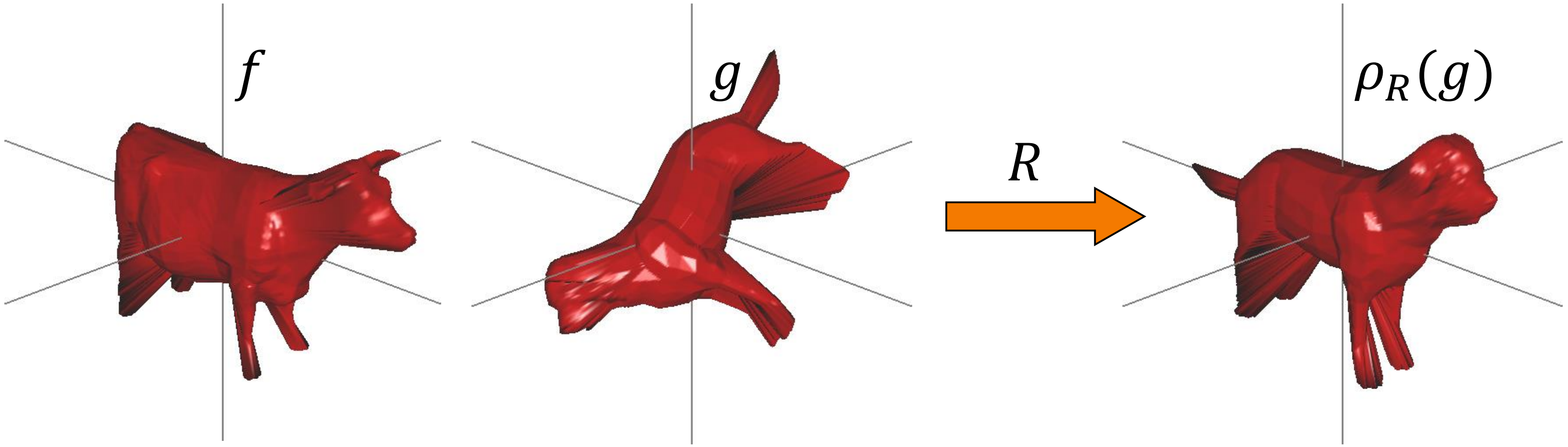
- Solving PDEs

Implications / Applications

Fast Correlation (Applications):

For the group of rotations in 3D, we have seen applications in:

- Shape Alignment

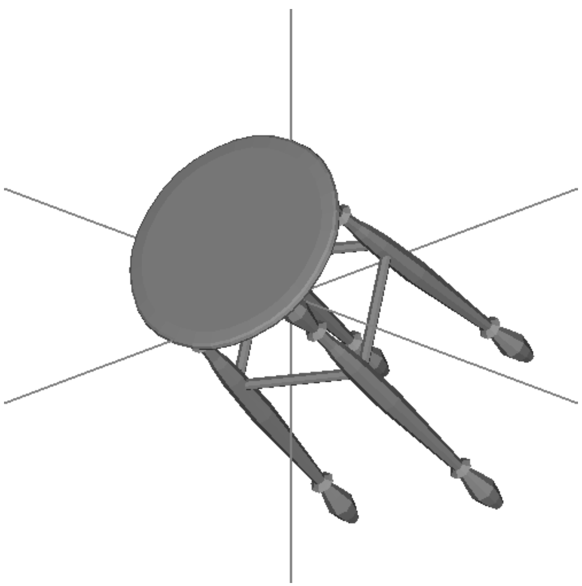


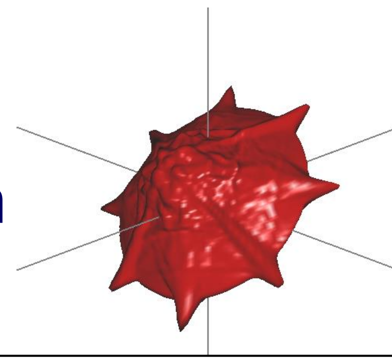
Implications / Applications

Fast Correlation (Applications):

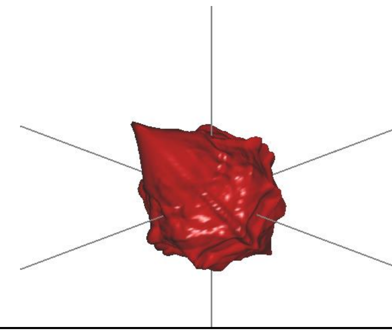
For the group of rotations in 3D, we have seen applications in:

- Shape Alignment
- Symmetry Detection

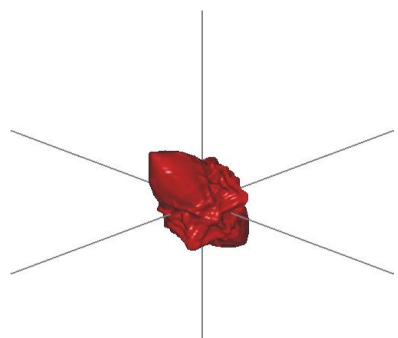




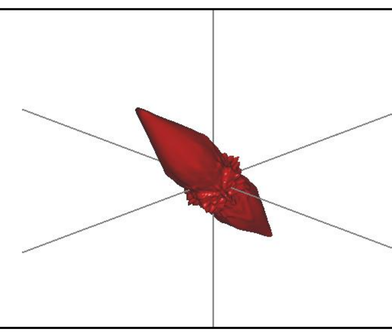
Reflective Symmetries



2-Fold
Rotational Symmetries

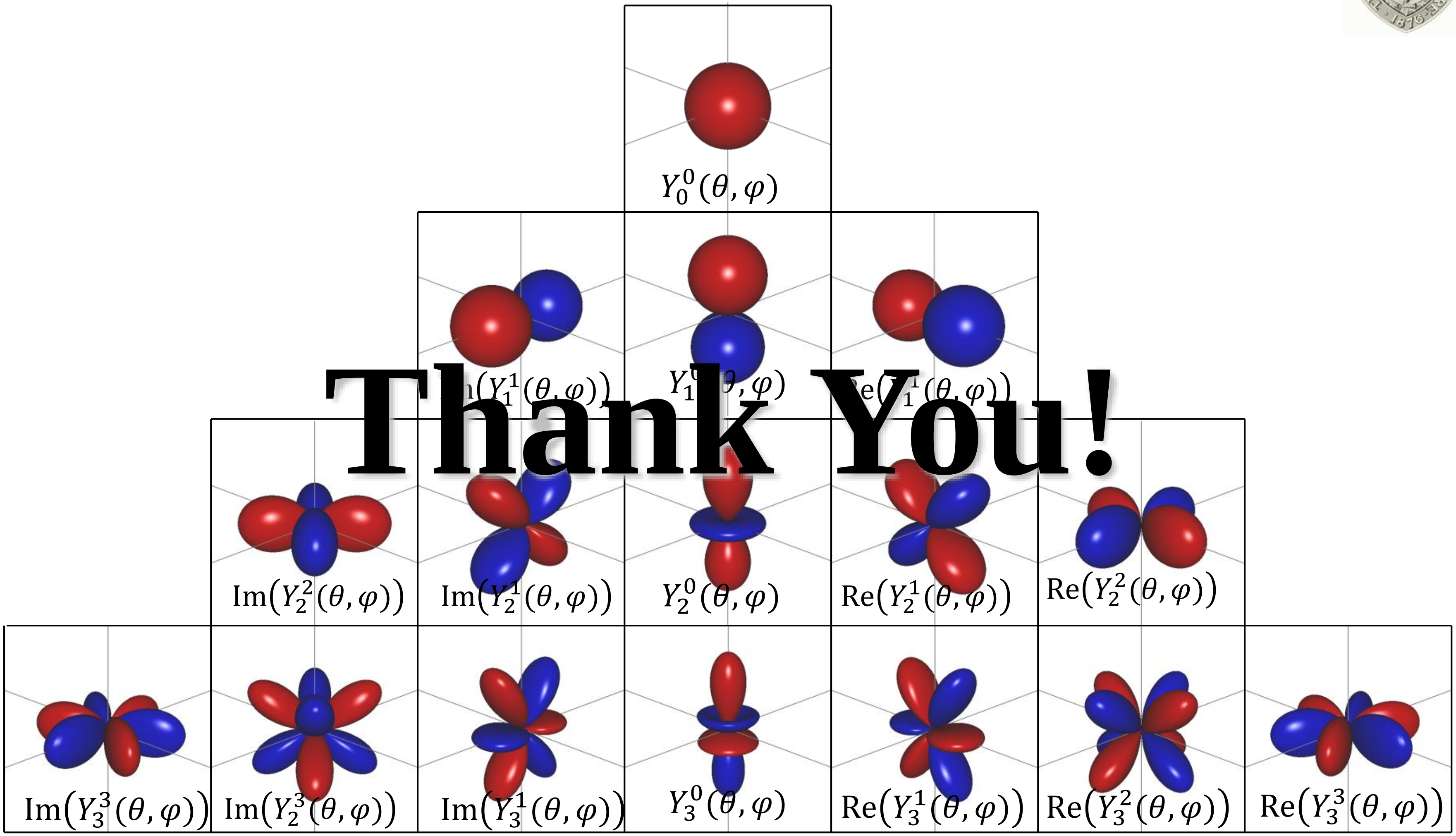


3-Fold
Rotational Symmetries



4-Fold
Rotational Symmetries

Thank You!



$Y_0^0(\theta,\varphi)$
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$\mathrm{Im}(Y_2^1(\theta,\varphi))$
$Y_2^0(\theta,\varphi)$
$\mathrm{Re}(Y_2^1(\theta,\varphi))$
$\mathrm{Re}(Y_2^2(\theta,\varphi))$
$\mathrm{Im}(Y_3^3(\theta,\varphi))$
$\mathrm{Im}(Y_2^3(\theta,\varphi))$
$\mathrm{Im}(Y_3^1(\theta,\varphi))$
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