Mesh

Data Representations

- List of vertices:
  - Pointer to a half edge sourced at the vertex.
- List of triangles
  - Pointer to half edge
- List of half edges:
  - Pointer to next half edge
  - Pointer to opposite half edge
  - Pointer to the incident face
  - Pointer to source vertex

What is this useful for?
Fast local queries!!
Mesh

Data Representations

• List of vertices
• List of triangles
  • Pointer to three vertices

What is this useful for?
Global computations!!
Functions on meshes

Defined at vertices, affine linear at faces
Functions on meshes

Integration is done per triangle element:

\[
\int_T f := \sum_{t_i \in T} \int_{t_i} f
\]

Exercise:

Given a triangle \( t \subset \mathbb{R}^3 \), with vertices \( v_0, v_1, v_2 \), and \( f: t \to \mathbb{R} \), affine linear function with \( f(v_0) = f_0, f(v_1) = f_1, f(v_2) = f_2 \)

Compute:
- \( \int_t f \) in terms of the triangle area \( A_t \) and \( f_0, f_1, f_2 \)
- \( \int_t f^2 \) in terms of the triangle area \( A_t \) and \( f_0, f_1, f_2 \)
For each vertex $i$ define $B_i$, the piecewise affine linear function, that has value 1 at vertex $i$ and 0 at the other vertices.

We can represent $f = \sum f_i B_i$

Given a triangle $t \subset \mathbb{R}^3$, with vertices $v_0, v_1, v_2$,

Compute

- $\int_t B_i B_j$ for $i, j \in \{0, 1, 2\}$ in terms of the triangle area.
Functions on meshes

The gradient of a piecewise affine linear function is piecewise constant.

Exercise:
What is the direction of $\nabla B_i$? What is its magnitude?
Find an expression for $\nabla B_i$ in terms of the opposite edge $\hat{e}_i$ and the triangle area $A_t$. 
Functions on meshes

The gradient of a piecewise affine linear function is piecewise constant.

Exercise:

Prove that

\[ \nabla B_i + \nabla B_j + \nabla B_k = 0 \]

Prove that

\[ \int_t \langle \nabla B_i, \nabla B_j \rangle = -\frac{\cot(\alpha_{ij})}{2} \]