# Introduction to Computational Geometry

Michael Kazhdan

(600.459)

# What are we studying?

- Algorithms and data-structures for solving geometric problems.
- Following O'Rourke, "Computational Geometry in C (2nd Edition)":
  http://cs.smith.edu/~orourke/books/compgeom.html
  http://cs.smith.edu/~orourke/books/ftp.html

# More Specifically

- Polygon Triangulation
- Polygon Partitioning
- Convex Hulls in 2D/3D
- Voronoi Diagrams
- Arrangements
- Search & Intersection
- Motion Planning

# Additional Resources

QHull: www.qhull.org/

ANN: www.cs.umd.edu/~mount/ANN/

Triangle: www.cs.cmu.edu/~quake/triangle.html

TetGen: wias-berlin.de/software/tetgen/

CGAL: www.cgal.org/

# Syllabus

When: MW @ 1:30 – 2:45

Where: Maryland 214

Web: http://www.cs.jhu.edu/~misha/Spring16/

TA: Fabian Prada

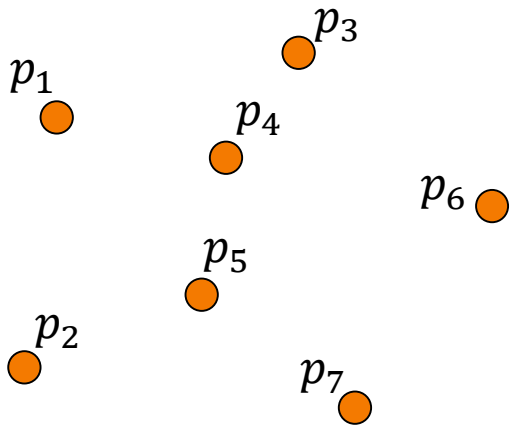
Assignments: Yes

Exams: Maybe

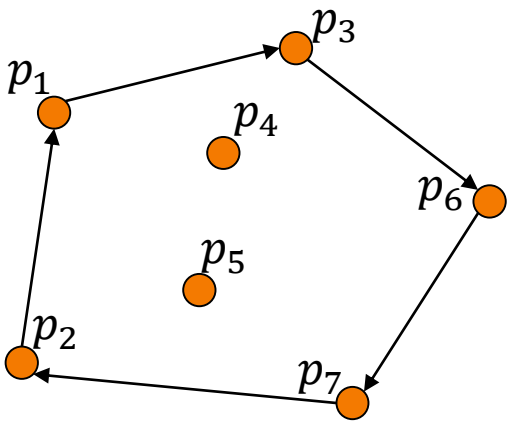
# Example: Convex Hulls in 2D

Given: A set $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$.

Output: An ordered subset $CH = \{p_{i_1}, \ldots, p_{i_m}\} \subset P$ describing the convex hull.




$P$




$CH = \{p_1, p_3, p_6, p_7, p_2\}$

# Example: Convex Hulls in 2D

Take 1:

1. --- $E \leftarrow \emptyset$
2. --- $\forall (p, q) \in P \times P$ with $p \neq q$
3. ------ $valid \leftarrow true$
4. ------ $\forall r \in P$ with $r \neq p$ and $r \neq q$
5. -------- If $r$ is to the left of the $\overrightarrow{pq}$: $valid \leftarrow false$
6. ------ If $valid$: $E \leftarrow E \cup \overrightarrow{pq}$
7. --- Construct $CH$ from $E$.

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Correctness?
Robustness?
Efficiency?

# Example: Convex Hulls in 2D

Take 2: (Graham's Scan)

1. --- Sort $P$ lexicographically
2. --- $L_{upper} \leftarrow \{p_1, p_2\}$
3. --- for $i \in \{3, \ldots, n\}$
4. ------ $L_{upper} \leftarrow L_{upper} \cup \{p_i\}$
5. ------ While $|L_{upper}| > 2$ and the last three points don't make a right turn:
6. --------- Delete the middle of the last three points
7. --- Repeat above for $L_{lower}$, working backward from $p_n$
8. --- $CH = L_{upper} \cup L_{lower}$

# For Next Class (Monday)

Chapter 1: Polygon Triangulation