Introduction to Computational Geometry

Michael Kazhdan

(600.459)
What are we studying?

• Algorithms and data-structures for solving geometric problems.

• Following O’Rourke, “Computational Geometry in C (2nd Edition)”:
  http://cs.smith.edu/~orourke/books/compgeom.html
  http://cs.smith.edu/~orourke/books/ftp.html
More Specifically

- Polygon Triangulation
- Polygon Partitioning
- Convex Hulls in 2D/3D
- Voronoi Diagrams
- Arrangements
- Search & Intersection
- Motion Planning
Additional Resources

QHull: www.qhull.org/

ANN: www.cs.umd.edu/~mount/ANN/

Triangle: www.cs.cmu.edu/~quake/triangle.html

TetGen: wias-berlin.de/software/tetgen/

CGAL: www.cgal.org/
Syllabus

When: MW @ 1:30 – 2:45
Where: Maryland 214
Web: http://www.cs.jhu.edu/~misha/Spring16/
TA: Fabian Prada
Assignments: Yes
Exams: Maybe
Example: Convex Hulls in 2D

**Given**: A set $P = \{p_1, \ldots, p_n\} \subset \mathbb{R}^2$.

**Output**: An ordered subset $CH = \{p_{i_1}, \ldots, p_{i_m}\} \subset P$ describing the convex hull.

$CH = \{p_1, p_3, p_6, p_7, p_2\}$
Example: Convex Hulls in 2D

Take 1:

1. --- $E \leftarrow \emptyset$

2. --- $\forall (p, q) \in P \times P$ with $p \neq q$

3. ------ $valid \leftarrow true$

4. ------ $\forall r \in P$ with $r \neq p$ and $r \neq q$

5. -------- If $r$ is to the left of the $pq$: $valid \leftarrow false$

6. ------ If $valid$: $E \leftarrow E \cup \overrightarrow{pq}$

7. --- Construct $CH$ from $E$. 
Example: Convex Hulls in 2D

Take 1:

1. \( E \leftarrow \emptyset \)
2. \( \forall (p, q) \in P \times P \) with \( p \neq q \)
3. \( \text{valid} \leftarrow \text{true} \)
4. \( \forall r \in P \) with \( r \neq p \) and \( r \neq q \)
5. \( \text{If } r \text{ is to the left of the } \overrightarrow{pq} \): \( \text{valid} \leftarrow \text{false} \)
6. \( \text{If } \text{valid}: \ E \leftarrow E \cup \overrightarrow{pq} \)
7. \( \text{Construct } CH \text{ from } E \).

Correctness? Robustness? Efficiency?
Example: Convex Hulls in 2D

**Take 2: (Graham’s Scan)**

1. --- Sort $P$ lexicographically
2. --- $L_{upper} \leftarrow \{p_1, p_2\}$
3. --- for $i \in \{3, \ldots, n\}$
4. ------ $L_{upper} \leftarrow L_{upper} \cup \{p_i\}$
5. ------ While $|L_{upper}| > 2$ and the last three points don’t make a right turn:
6. ------- --- Delete the middle of the last three points
7. --- Repeat above for $L_{lower}$, working backward from $p_n$
8. --- $CH = L_{upper} \cup L_{lower}$
For Next Class (Monday)

Chapter 1: Polygon Triangulation