Image Processing

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HB Ch. 14.4
FvDFH Ch. 13.1
Outline

• Human Vision
• Image Representation
• Reducing Color Quantization Artifacts
• Basic Image Processing
Human Vision

Model of Human Visual System

Sun

Objects in world

Human eye
Electromagnetic Spectrum

- Visible light frequencies range between ...
  - Red = $4.3 \times 10^{14}$ hertz (700nm)
  - Violet = $7.5 \times 10^{14}$ hertz (400nm)

Figures 15.1 from H&B
Visible Light

- The human eye can “see” light in the frequency range 400nm – 700nm

**White Light**

Figure 15.3 from H&B
Human Vision

The human retina contains two types of photoreceptors, **cones** and **rods**.

**Cones:**
- 6-7 million cones in the retina
- Responsible for photopic vision
- Color sensitive:
  - 64% red, 32% green, 2% blue
- Distributed in the fovea centralis

**Rods:**
- 120 million rods in the retina
- 1000x more light sensitive than cones
- Responsible for scotopic vision
- Short-wavelength sensitive
- Responsible for peripheral vision
Tristimulus Theory of Color

Spectral-response functions of each of the three types of cones on the human retina.

This motivates encoding color as a combination of red, green, and blue (RGB).

Figure 13.18 from FvDFH
Visible Light

- The human eye can “see” light in the frequency range 400nm – 700nm

This does not mean that we can see the difference between the different spectral distributions.

Metamers = Two spectral distributions that look the same

White Light

Figure 15.3 from H&B
Outline

• Human Vision
• Image Representation
• Reducing Color Quantization Artifacts
• Basic Image Processing
Image Representation

What is an image?
Image Representation

An image is a 2D rectilinear array of pixels:

A \textit{width} \times \textit{height} array where each entry of the array stores a single pixel.

Continuous image

Digital image
Image Representation

What is a pixel?

Continuous image

Digital image
Image Representation

A pixel is something that captures the notion of color.

- **Luminance pixels**
  - Grey-scale images (aka “intensity images”)
  - 0 – 1.0 or 0 – 255

- **Red, Green, Blue pixels (RGB)**
  - Color images
  - 0 – 1.0 or 0 – 255
Resolutions

- Spatial resolution: \textit{width} \times \textit{height} pixels
- Intensity/Color resolution: \( n \) bits per pixel
- Temporal resolution: \( n \) Hz (fps)

<table>
<thead>
<tr>
<th></th>
<th>width \times height</th>
<th>bit depth</th>
<th>Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handheld</td>
<td>2220 \times 1080</td>
<td>24</td>
<td>60</td>
</tr>
<tr>
<td>Monitor</td>
<td>3840 \times 1080</td>
<td>24</td>
<td>144</td>
</tr>
<tr>
<td>CCDs</td>
<td>6000 \times 4000</td>
<td>36</td>
<td>50</td>
</tr>
<tr>
<td>Laser Printer</td>
<td>6600 \times 5100</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>
Image Quantization Artifacts

• With only a small number of bits associated to each color channel of a pixel there is a limit to intensity resolutions of an image
  ◦ A black and white image allocates a single bit to the luminance channel of a pixel.
    » The number of different colors that can be represented by a pixel is 2.
  ◦ A 24 bit bitmap image allocates 8 bits to the red, green, and blue channels of a pixel.
    » The number of different colors that can be represented by a pixel is 16,000,000.
Outline

• Human Vision

• Image Representation

• Reducing Color Quantization Artifacts
  ◦ Halftoning and Dithering

• Basic Image Processing
Reducing Color Quantization Artifacts

Key Idea:

For (still) images, the combination of image resolution and intensity/color resolution define the total informational content.

We can trade off between these to achieve different visual effects.
Disclaimer:

In the next few slides, we assume that the original image has continuous pixel values, $I(x, y) \in [0,1)$.

In practice, all the images you will work with will have integer values, $I(x, y) \in \{0, \ldots, 255\}$.

\[ [0,1) \] is the set of numbers between 0 and 1 including 0 but not including 1.
Quantization

- When you have a small number of bits per pixel, you can coarsely represent an image by quantizing the color values:

\[ P(x, y) = Q_b(I(x, y)) = \text{floor}(I(x, y) \cdot 2^b) \]

with \( b \) the number of bits per pixel.
Quantization

Image with decreasing bits per pixel

- Note contouring!

\[ b = 8 \text{ bits} \]

\[ b = 4 \text{ bits} \]

\[ b = 2 \text{ bits} \]

\[ b = 1 \text{ bits} \]
Reducing Effects of Quantization

Trade spatial resolution for intensity resolution:

• Half-toning
• Dithering
Classical Half-Toning

- Varying-size dots represent intensities
- Area of dots inversely proportional to intensity

\[ I(x, y) \]

\[ P(x, y) \]
Classical Half-Toning

Newspaper Image

From New York Times, 9/21/99
Digital Half-Toning

• Use cluster of pixels to draw (average) intensity
• Trades spatial resolution for intensity resolution
• Note:
  ◦ Half-toning pattern matters
    » Want to avoid vertical, horizontal lines
  ◦ Loss of information
    » 16 configurations → 5 intensities

\[
0 \leq I \leq 0.2 \quad 0.2 \leq I \leq 0.4 \quad 0.4 \leq I \leq 0.6 \quad 0.6 \leq I \leq 0.8 \quad 0.8 \leq I \leq 1
\]
Digital Half-Toning

- Use cluster of pixels to draw
- Trades spatial resolution for intensity resolution
- Note:
  - Half-toning pattern matters
    - Want to avoid vertical, horizontal lines
  - Loss of information
    - 16 configurations → 5 intensities

Original (8 bits)
Quantized (1 bit)
Half-toned (1 bit)
Dithering

- Distribute errors among pixels
  - Exploit spatial integration in our eye
  - Display greater range of perceptible intensities
Random Dither

- Randomize quantization errors
- Errors appear as noise

If a pixel is black, then adding random noise to it, you are less likely to turn it into a white pixel than if the pixel were dark gray.

\[
P(x, y) = Q_b \left( I(x, y) + \frac{\text{noise}(x, y)}{2^b} \right)
\]
Random Dither

- Randomize quantization errors
- Errors appear as noise

How much noise should we add?

Enough so that we effect rounding, but not so much that we overshoot:

$(-1.0,1.0)$

$$P(x, y) = Q_b \left( I(x, y) + \frac{\text{noise}(x, y)}{2^b} \right)$$
Random Dither

Original (8 bits)  Uniform (1 bit)  Random (1 bit)
Ordered Dither

Similar to quantization:
   We round the input to a value in the range \( \{0, \ldots, 2^b - 1\} \).

Different from quantization:
   How we round depends on the pixel’s spatial position.
Ordered Dither (Binary Displays)

- Pseudo-random quantization errors
- $n \times n$ matrix stores pattern of thresholds

// Locate the index in the matrix:
\[ i = x \mod n \]
\[ j = y \mod n \]

// Get fractional component
\[ e = I(x, y) \]

// Round up/down
\[
\begin{align*}
\text{if } (e > \frac{D_n(i,j)}{n^2+1}) & \\ P(x, y) &= 1 \\
\text{else} & \\
P(x, y) &= 0
\end{align*}
\]

\[
D_2 = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}
\]
Ordered Dither (*b*-Bit Displays)

- Pseudo-random quantization errors
- \( n \times n \) matrix stores pattern of thresholds

```plaintext
// Locate the index in the matrix:
i = x \mod n
j = y \mod n

// Get fractional component
\[ c = I(x, y) \cdot (2^b - 1) \]
\[ e = c - \text{floor}(c) \]

// Round up/down
if \( e > \frac{D_n(i,j)}{n^2+1} \)
P(x, y) = \text{ceil}(c)
else
P(x, y) = \text{floor}(c)
```

\[ D_2 = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix} \]
Ordered Dither

Original (8 bits)

Uniform (1 bit)

Random (1 bit)

Ordered (1 bit)
Error Diffusion Dither

• Spread quantization error over neighbor pixels
  ◦ Error dispersed to pixels right and below

• Floyd-Steinberg Method

\[
\alpha + \beta + \gamma + \delta = 1.0
\]

Figure 14.42 from H&B
Error Diffusion Dither

\[
\text{for( } j=0 \text{ ; } j<\text{height} \text{ ; } j++ \) \\
\text{for( } i=0 \text{ ; } i<\text{width} \text{ ; } i++ \)
\]
\[
\text{Dest}_{i,j} = \text{quantize( Source}_{i,j} \) \\
\text{error} = \text{Source}_{i,j} - \text{Dest}_{i,j} \\
\text{Source}_{i,j+1} = \text{Source}_{i,j+1} + \alpha \times \text{error} \\
\text{Source}_{i+1,j-1} = \text{Source}_{i+1,j-1} + \beta \times \text{error} \\
\text{Source}_{i+1,j} = \text{Source}_{i+1,j} + \gamma \times \text{error} \\
\text{Source}_{i+1,j+1} = \text{Source}_{i+1,j+1} + \delta \times \text{error}
\]

\[
\alpha = \frac{7}{16} \quad \beta = \frac{3}{16} \quad \gamma = \frac{5}{16} \quad \delta = \frac{1}{16}
\]

Floyd-Steinberg Dither
Error Diffusion Dithering

- Original (8 bits)
- Uniform (1 bit)
- Random (1 bit)
- Ordered (1 bit)
- Floyd-Steinberg (1 bit)
Outline

• Human Vision
• Image Representation
• Reducing Color Quantization Artifacts
• Basic Image Processing
  ◦ Single Pixel Operations
Computing Grayscale

• The human retina perceives red, green, and blue as having different levels of brightness.

• To compute the luminance (perceived brightness) of a pixel, we need to take the weighted average of the RGBs:
  \[ L_p = 0.30 \cdot r_p + 0.59 \cdot g_b + 0.11 \cdot b_p \]
Adjusting Brightness

• Scale pixel components
  ◦ Must clamp to range (e.g., 0 to 255)

\[ I_p \leftarrow I_p \cdot \alpha \]
Adjusting Contrast

- Compute mean image luminance $\bar{L}$
  - $\bar{L} = \text{Average}(0.30 \cdot r_p + 0.59 \cdot g_p + 0.11 \cdot b_p)$

- Scale deviation from $\bar{L}$ for each pixel component
  - Must clamp to range (e.g., 0 to 255)

\[ I_p \leftarrow (I_p - \bar{L})\alpha + \bar{L} \]
Adjusting Contrast

• Compute mean image luminance $\bar{L}$
  $\bar{L} = \text{Average}(0.30 \cdot r_p + 0.59 \cdot g_p + 0.11 \cdot b_p)$

What happens if we set the image to have no contrast ($\alpha = 0$)?

Original

More Contrast

$I_p \leftarrow (I_p - \bar{L})\alpha + \bar{L}$
Adjusting Saturation

• Compute per-pixel luminance \( L_p \)
  \[
  L_p = 0.30 \cdot r_p + 0.59 \cdot g_p + 0.11 \cdot b_p
  \]

• Scale deviation from \( L_p \) for each pixel component
  ○ Must clamp to range (e.g., 0 to 255)

\[
I_p \leftarrow (I_p - L_p)\alpha + L_p
\]
Adjusting Saturation

- Compute per-pixel luminance $L_p$
  \[ L_p = 0.30 \cdot r_p + 0.59 \cdot g_p + 0.11 \cdot b_p \]

What happens if we set the image to have no saturation ($\alpha = 0$)?

Original → More Saturation

\[ I_p \leftarrow (I_p - L_p)\alpha + L_p \]