Radiosity

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Announcement

• Midterm (December 6\textsuperscript{th})

• Assignment 4 (December 14\textsuperscript{th})
  ◦ Due December 14\textsuperscript{th}
  ◦ No possibility of extensions (beyond late days)
Overview

• Ray Tracing Revisited

• Radiosity
Ray Casting

Ray tracing is based on the Phong lighting model:

- A surface reflects light non-uniformly, with stronger reflection in the specular direction:

\[ I = I_E + K_A I_A + \sum_L (K_D \langle N, L \rangle + K_S \langle V, R \rangle^n) I_L S_L \]
Ray Casting

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Specular Contribution
Specular Lobe
Ray Tracing

Ray tracing is based on the Phong lighting model:

As a result, when we cast secondary rays, we cast them in the reflected direction to maximize the contribution to the lighting computation.

\[ I = I_E + K_A I_A + \sum_L (K_D \langle N, L \rangle + K_S \langle V, R \rangle^n) I_L S_L + K_S I_R \]
Ray Tracing

Advantage:

- It does a good job of capturing the specular properties of materials.
Ray Tracing

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- It does a good job of capturing the specular properties of materials.

Disadvantages:
- Difficult to support soft shadows from area lights
- Difficult to support caustics
- Need the ambient term as a hack for the global illumination.
Lighting

What do we really want to compute?

The accumulation of light coming in from all directions, modulated by how much the light is reflected in/from that direction.
Lighting

What do we really want to compute?

The brightness of the light that reaches the camera from some point \( p \) in the scene is the sum:

- Of the light emitted from \( p \), to the camera plus the sum:
  - Of the light emanating from every point in the scene,
  - Scaled by the extent to which is reflected through \( p \).
Lighting

What do we really want to compute?

Light emitted from $p$ to $e$

Light arriving at $e$ from $p$

Fraction of light arriving from $s$ that is reflected towards $e$

Amount of light arriving at $p$ from $s$

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_{S} F_r(s \rightarrow p \rightarrow e) B(s \rightarrow p) ds$$
Lighting

Challenge:

- The integral needs to be computed over a large number of points because the function is discontinuous.
- The function is recursive since the amount of light leaving a point depends on the amount of light entering it.

\[
B(p \rightarrow e) = E(p \rightarrow e) + \int_S F_r(s \rightarrow p \rightarrow e) B(s \rightarrow p) ds
\]
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions.
  - Lights: modeled by uniformly emissive surfaces.

\[ B(p \rightarrow e) = E(p \rightarrow e) + \int_S F_r(s \rightarrow p \rightarrow e)B(s \rightarrow p)ds \]
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions
  - Lights: modeled by uniformly emissive surfaces.
  - Objects: surface are Lambertian

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_S F_r(s \rightarrow p \rightarrow e)B(s \rightarrow p)ds$$
Lambertian Lighting

Assuming the receiver is directed at $s$:

The perceived brightness is independent of the viewer’s position with respect to the surface.

Under our assumptions, point $s$ is equally bright to $p_1$ and $p_2$. 
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However, a patch of size $A$ about $p_1$ gets contributions from a patch of size $A / \cos \theta_1$ about $s$. 
Lambertian Lighting

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The perceived brightness is independent of the viewer’s position with respect to the surface.

Under our assumptions, point $s$ is equally bright to $p_1$ and $p_2$.

However, a patch of size $A$ about $p_1$ gets contributions from a patch of size $A/\cos \theta_1$ about $s$.

Thus, under our assumptions, the amount of light $s$ emits/reflects to a point $p$ falls off as $\cos \theta$. 
Lambertian Lighting

If the receiver is not directed at $s$:

The amount of light falling on a unit patch at point $p$ will depend on the angle between the line from $p$ to $s$ and the normal at $p$.

The perceived brightness (per unit area) at a point $p$ falls off as $\cos \theta$. 

\[
\begin{align*}
A & \cos \theta_1 \\
A & \cos \theta_2
\end{align*}
\]
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions
  - Lights: modeled by uniformly emissive surfaces.
  - Objects: surfaces are Lambertian

So how does this affect the lighting equation?
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at $p$ is constant in all directions.

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_S F_r(s \rightarrow p \rightarrow e)B(s \rightarrow p)ds$$
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at \( p \) is constant in all directions.

\[
B(p \rightarrow e) = E(p) + \int_S F_r(s \rightarrow p \rightarrow e)B(s \rightarrow p)ds
\]
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at $p$ is constant in all directions.
- The perceived brightness of a light is independent of the viewer’s position.

$$B(p \rightarrow e) = E(p) + \int_S F_r(s \rightarrow p \rightarrow e) B(s \rightarrow p) ds$$
Radiosity

So how does this affect the lighting equation?

- The perceived brightness of an emitting surface at \( p \) is constant in all directions.
- The perceived brightness of a light is independent of the viewer’s position.

\[
B(p) = E(p) + \int_{s} F_r(s \rightarrow p) B(s) \, ds
\]
Radiosity

The fraction of light from point $s$ that is reflected off of $p$ is determined by:

- The visibility of $p$ from $s$: $V(s, p)$
- The angle: $\theta_{s,p}$
- The angle: $\theta_{p,s}$
- The distance from $s$ to $p$: $\|s - p\|$
- The material properties at $p$: $\rho(p)$

$$F_r(s \rightarrow p) = \rho(p)V(s, p) \frac{\cos \theta_{s,p} \cdot \cos \theta_{p,s}}{\|s - p\|^2}$$

$$B(p) = E(p) + \int_{s} F_r(s \rightarrow p) B(s) ds$$
Radiosity

Make simplifying assumptions about the scene:

- Perceived brightness is equal in all directions
  - Lights: modeled by uniformly emissive surfaces.
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\[ B(p) = E(p) + \rho(p) \int_V V(s, p) \frac{\cos \theta_{s,p} \cdot \cos \theta_{p,s}}{||s - p||^2} B(s)ds \]

The radiosity equation
Radiosity

Approximate the solution by decomposing surfaces into patches and doing a discrete summation:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

Form Factor

\[ B(p) = E(p) + \rho(p) \int_s V(s, p) \frac{\cos \theta_{s,p} \cdot \cos \theta_{p,s}}{\|s - p\|^2} B(s)ds \]

The radiosity equation
Form Factor

The form factor $F_{j,i}$ is the proportion of the total power leaving patch $P_j$ that is received by patch $P_i$:

- $A_i F_{i,j} = A_j F_{j,i}$
- $F_{j,i} \geq 0$
- $\sum_i F_{j,i} \leq 1$
- $F_{ii} = 0$ unless the patch is concave
Radiosity

Approximate the solution by decomposing surfaces into patches and doing a discrete summation:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

This amounts to solving a linear system of equations:
- \( E_i, \rho_i, \) and \( F_{j,i} \) are given,
- \( B_i \) are the unknowns.
Radiosity

Re-ordering terms in the equation gives:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

\[ \Downarrow \]

\[ E_i = B_i - \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

\[ \begin{pmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{pmatrix} = \begin{pmatrix} 1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{1,2} & \cdots & -\rho_1 \cdot F_{1,n} \\ -\rho_2 \cdot F_{2,1} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ -\rho_n \cdot F_{n,1} & -\rho_n \cdot F_{n,2} & \cdots & 1 - \rho_n \cdot F_{n,n} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{pmatrix} \]
Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors

\[
\begin{pmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{pmatrix} =
\begin{pmatrix}
1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{1,2} & \cdots & -\rho_1 \cdot F_{1,n} \\
-\rho_2 \cdot F_{2,1} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n \cdot F_{n,1} & -\rho_n \cdot F_{n,2} & \cdots & 1 - \rho_n \cdot F_{n,n}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{pmatrix}
\]

\[
\hat{e} = A \cdot \vec{b}
\]
Solving the System of Equations

• Solution methods:
  ◦ Invert the matrix \( O(n^3) \)
  ◦ Iterative methods – \( O(n^2) \)
  ◦ Progressive methods – \(< O(n^2)\)

\[
\begin{pmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{pmatrix} =
\begin{pmatrix}
1 - \rho_1 \cdot F_{1,1} & -\rho_1 \cdot F_{1,2} & \cdots & -\rho_1 \cdot F_{1,n} \\
-\rho_2 \cdot F_{2,1} & 1 - \rho_2 \cdot F_{2,2} & \cdots & -\rho_2 \cdot F_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_n \cdot F_{n,1} & -\rho_n \cdot F_{n,2} & \cdots & 1 - \rho_n \cdot F_{n,n}
\end{pmatrix}
\begin{pmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{pmatrix}
\]

\[
\hat{\vec{e}} = A \cdot \vec{b}
\]
Gauss-Seidel Iteration

Initialization:

- For each patch $P_i$, initialize its radiosity to be equal to its emissiveness:

\[ B_i = E_i \]
Gauss-Seidel Iteration

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- For each patch $P_i$, initialize its radiosity to be equal to its emissiveness:
  \[ B_i = E_i \]

Iteration:

- At each iteration, update the values of each of the $B_i$ based on the values of all the other $B_j$:
  \[ B_i = E_i + \rho_i \sum_{j \neq i} F_{j,i} \cdot B_j \]
Gauss-Seidel Iteration

- Geometric interpretation
  - Iteratively gather radiosity to elements
Gauss-Seidel Iteration

- Geometric interpretation
  - Iteratively gather radiosity to elements

Limitation:

Can spend a lot of time gathering radiosity from patches that don’t contribute much.
Progressive Radiosity

- Geometric interpretation:
  - Iteratively shoot “unshot” radiosity from elements
  - Select shooters in order of unshot radiosity
Progressive Radiosity

- Progressive refinement
Summary

If we could, we would compute the lighting by recursively reflecting secondary rays in all directions to compute the brightness of a single point.

- Ray-Tracing:
  - Assume that surfaces are specular so that you only need to bounce in a single (specular) direction.

- Radiosity:
  - Assume that surfaces are Lambertian so that they reflect light in the same way in all directions.

- Reality:
  - Surfaces reflect in all directions, but not uniformly.