Shading and Visibility

Michael Kazhdan

(601.457/657)

HB 13.2 -- 13.8, 14.5
FvDFH 15.4, 15.5, 15.6, 15.7.1, 16.2
3D Rendering Pipeline (for direct illumination)

3D Primitives

Modeling Transformation

3D Modeling Coordinates

Camera Transformation

3D World Coordinates

Lighting

3D Camera Coordinates

Projection Transformation

3D Camera Coordinates

Clipping

2D Screen Coordinates

Viewport Transformation

2D Screen Coordinates

Scan Conversion

2D Image Coordinates

Image

2D Window

3D Model

2D Screen
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- 3D Primitives
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3D Primitives → 3D Modeling Coordinates

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Scan Conversion → Image

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Overview

• Scan conversion
  ○ Figure out which pixels to fill

• Shading
  ○ Determine a color for each filled pixel

• Depth test
  ○ Determine when the color of a pixel comes from the front-most primitive
Polygon Shading

• Simplest shading approach is to perform independent lighting calculation for every pixel

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right) \]
Polygon Shading

- Can take advantage of spatial coherence
  - Illumination calculations for pixels covered by same primitive are related to each other

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + K_S \langle \hat{V}, \hat{R}_i \rangle^n I_i \right) \]
Polygon Shading Algorithms

- Flat Shading
- Gouraud Shading
- Phong Shading
Flat Shading

- Can take advantage of spatial coherence
  - Make the lighting equation constant over the surface of each primitive

<table>
<thead>
<tr>
<th></th>
<th>Surface Normal</th>
<th>Light Direction</th>
<th>View Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissive</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Ambient</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Diffuse</td>
<td>+</td>
<td>+</td>
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\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L_i} \rangle I_i + K_S \langle \vec{V}, \vec{R_i} \rangle^i I_i \right) \]
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</tr>
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If the normal is constant over the primitive, and if the light is directional, the diffuse component is the same for all points on the primitive.

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right)
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If the normal is constant over the primitive, and if the light is directional, the diffuse component is the same for all points on the primitive.

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<td></td>
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\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + K_S \langle \hat{V}, \hat{R}_i \rangle^n I_i \right)
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### Flat Shading

- Can take advantage of spatial coherence
  - Make the lighting equation constant over the surface of each primitive

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<tr>
<td></td>
<td>and if parallel projection is used the specular component is the same for all points on the primitive</td>
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I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle n_i I_i \right)
\]
Flat Shading

- Illuminate as though all light sources are directional, the polygon is flat, and the camera uses parallel projection
  - $\langle \hat{N}, \hat{L}_i \rangle$ constant over surface
  - $\langle \hat{V}, \hat{R}_i \rangle$ constant over surface
  - $I_i$ constant over surface

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + K_S \langle \hat{V}, \hat{R}_i \rangle n I_i \right)
\]
Flat Shading

- One lighting calculation per polygon
  - Assign all pixels inside each polygon the same color
Flat Shading

- Objects look like they are composed of polygons
  - OK for polyhedral objects
  - Not so good for smooth surfaces
Polygon Shading Algorithms

- Flat Shading
- **Gouraud Shading**
- Phong Shading
Gouraud Shading

- What if smooth surface is represented by polygonal mesh with a normal at each vertex?

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right) \]
Gouraud Shading

- One lighting calculation per vertex
  - Assign pixel colors inside polygon by interpolating colors computed at vertices
Gouraud Shading

- Linearly interpolate colors at vertices down and across scan lines

\[ A = (1 - \alpha) \cdot I_1 + \alpha \cdot I_2 \]
\[ B = (1 - \beta) \cdot I_2 + \beta \cdot I_3 \]
\[ I = (1 - \gamma) \cdot A + \gamma \cdot B \]
Gouraud Shading

- Linearly interpolate colors at vertices down and across scan lines

\[ A = (1 - \alpha) \cdot I_1 + \alpha \cdot I_2 \]
\[ B = (1 - \beta) \cdot I_2 + \beta \cdot I_3 \]

Note: The values of \( \alpha \) and \( \beta \) only need to be updated as we move to the next scan-line. The value of \( \gamma \) needs to be updated as we advance along the scan-line.
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Continuous shading over adjacent polygons

Flat Shading

Gouraud Shading
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Continuous shading over adjacent polygons

What happens with large polygon & spotlight?
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Continuous shading over adjacent polygons

What happens with large polygon & spotlight?
Polygon Shading Algorithms

• Flat Shading
• Gouraud Shading
• Phong Shading
Phong Shading

- One lighting calculation per pixel
  - Approximate surface normals for points inside polygons by linear interpolation of normals from vertices

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right)
\]
Phong Shading

- Linearly interpolate surface normals at vertices down and across scan lines

\[
\vec{A} = (1 - \alpha) \cdot \vec{N}_1 + \alpha \cdot \vec{N}_2
\]
\[
\vec{B} = (1 - \beta) \cdot \vec{N}_2 + \beta \cdot \vec{N}_3
\]
\[
\vec{N} = (1 - \gamma) \cdot \vec{A} + \gamma \cdot \vec{B}
\]
Phong Shading

• Linearly interpolate surface normals at vertices down and across scan lines
Phong Shading

- Linearily interpolate surface normals at vertices down and across scan lines

This was not supported in early generation graphic cards but can now be implemented in the fragment shader.
Polygon Shading Algorithms

Wireframe

Flat

Gouraud

Phong
**3D Rendering Pipeline** *(for direct illumination)*

- **3D Primitives**
- **Modeling Transformation** → **3D Modeling Coordinates**
- **Camera Transformation** → **3D World Coordinates**
- **Camera Transformation** → **3D Camera Coordinates**
- **Lighting**
- **Projection Transformation** → **3D Camera Coordinates**
- **Projection Transformation** → **2D Screen Coordinates**
- **Clipping** → **2D Screen Coordinates**
- **Viewport Transformation** → **2D Screen Coordinates**
- **Scan Conversion** → **2D Image Coordinates**
- **Image**

---

**3D Model**

- 2D Window
- 2D Screen
Overview

• Scan conversion
  ◦ Figure out which pixels to fill

• Shading
  ◦ Determine a color for each filled pixel

• Depth test
  ◦ Determine when the color of a pixel comes from the front-most primitive
Hidden Surface Removal (HSR)

• Motivation

• Algorithms for HSR
  ◦ Back-face detection
  ◦ Depth sort
  ◦ Ray casting
  ◦ z-buffer
Motivation

In general, we don’t want to draw surfaces that are not visible to the viewer:

• Surfaces may be back-facing
• Surfaces may be covered in 3D
• Surfaces may be covered in the image plane
Somewhere in here we have to decide which objects are visible, and which objects are hidden.
Visibility algorithms

Figure 29. Characterization of ten opaque-object algorithms. A Comparison of the algorithms.

[Sutherland '74]
Overview

• Motivation

• Algorithms for HSR
  ○ Back-face detection
  ○ BSP-Trees
  ○ Ray casting
  ○ z-buffer
Q: How do we test for back-facing polygons?
A: Dot product of the normal and view directions.

If $\langle \vec{V}, \vec{N} \rangle > 0$, then polygon is back-facing.
Back-face detection

This method breaks down for:
• Overlapping primitives
• Non-solid models and/or models without a well defined orientation.

In general, back-face expected to remove \( \approx \) half of polygon surfaces from removal further visibility tests.
A polygon is back-facing if
\[ \langle \vec{V}, \vec{N} \rangle > 0 \iff \vec{N}_z > 0 \]
A polygon is back-facing if $\langle \vec{V}, \vec{N} \rangle > 0 \iff \vec{N}_z > 0$

**Note:** When your graphics card does this, it does **not** use the normals you provide at the vertices for lighting. Instead it uses the cross-product of the triangle vertices, so make sure that the ordering of the vertices is consistent (e.g. CCW)
View-frustrum culling

If the shape is outside the viewing volume, we don’t have to draw it.
View-frustrum culling

If the shape is outside the viewing volume, we don’t have to draw it.
View-frustrum culling

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  - Image

- 3D Modeling Coordinates
  - 3D World Coordinates
  - 3D Camera Coordinates
  - 2D Screen Coordinates
  - 2D Image Coordinates

- Trivial Reject

- Camera Transformation
- Modeling Transformation
- Projection Transformation
- Viewport Transformation
Ideal Solution

Painter’s Algorithm:

• Sort primitives front to back and draw the back ones first, over-writing pixel values with information from the front primitives as they are processed.

Problem:

• In general you can’t sort the primitives.
• ...Unless you are allowed to split them
**BSP-Tree Rendering**  *(Object Precision)*

- BSP-Trees recursively partition space by planes
  - Given two primitives on either side of a plane, the one on the opposite side from the camera will always be further away.
  - Draw the further side first, and then draw the closer one.
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
    - Draw right side of 3
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
    - Draw right side of 3
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
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    - Draw left side of 3
      - Draw D
    - Draw right side of 3
      - Draw left side of 5
      - Draw right side of 5
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

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  - Draw left side of 1
    - Draw left side of 2
    - Draw right side of 2
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      - Draw left side of 5
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        - Draw right side of 5
          - Draw F
  - Draw left side of 1
    - Draw left side of 2
      - Draw left side of 4
        - Draw right side of 4
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      - Draw left side of 4
        - Draw A
      - Draw right side of 4
        - Draw B
    - Draw right side of 2
      - Draw C
3D Rendering Pipeline

Binary Space Partition:
- View Independent
- Linear-time depth sort
Ray Casting

- Fire a ray for every pixel
  - If ray intersects multiple objects, take the closest
Ray Casting Pipeline

**Ray casting**
- $P(p \log n)$ for $p$ pixels and $n$ shapes
- May (or not) use pixel coherence
- Simple, but generally not used
**z-Buffer**

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Only update pixels whose depth is closer than the depth stored in the buffer

![Diagram of z-buffer with depth values](image)
z-Buffer

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Only update pixels whose depth is closer than the depth stored in the buffer

Case 1 (Blue before Red):
Blue $\rightarrow (d = 1) < (d = \infty)$: Set $RGB = (0,0,1), d = 1$
Red $\rightarrow (d = 2) > (d = 1)$: Don’t change pixel
z-Buffer

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- Blue $\rightarrow (d = 1) < (d = \infty)$:
  - Set $RGB = (0,0,1), d = 1$
- Red $\rightarrow (d = 2) > (d = 1)$:
  - Don’t change pixel

Case 2 (Red before Blue):
- Red $\rightarrow (d = 2) > (d = \infty)$:
  - Set $RGB = (1,0,0), d = 2$
- Blue $\rightarrow (d = 1) < (d = 2)$:
  - Set $RGB = (0,0,1), d = 1$
**z-Buffer**

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer
  - Depths are interpolated from vertices, just like colors

\[
A = (1 - \alpha) \cdot d_1 + \alpha \cdot d \\
B = (1 - \beta) \cdot d_2 + \beta \cdot d_3 \\
d = (1 - \gamma) \cdot A + \gamma \cdot B
\]
**z-Buffer**

Edge antialiasing becomes difficult because you want multiple triangles to write to the same pixel.

- Who sets the $z$-value?
**α-Buffer**

Alpha values can cause problems:

- Need to know the ordering of primitives behind pixels for $\alpha$-blending
- $\alpha$-buffer supports linked list of surfaces at each pixel for better transparency support
- $\alpha$-buffer also helps with anti-aliasing

$d = 1$

$d = 2$
3D Rendering Pipeline

- Polygons can be rasterized in any order
- Requires additional memory
  - \( z \)-buffer size \( \approx \) frame buffer
- This is what your graphics card does!
3D Rendering Pipeline (for direct illumination)

1. **3D Primitives**
2. **Modeling Transformation**
   - 3D Modeling Coordinates
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7. **Viewport Transformation**
   - 2D Screen Coordinates
8. **Scan Conversion**
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9. **Image**
   - 2D Image Coordinates

**Diagram Notes:**
- **3D Model**
- **2D Window**
- **2D Screen**
Scan Conversion

How do we average information from the three vertices of a triangle?

- Interpolate using weights determined by the 2D screen space projection.
- Interpolate using weights determined by the 3D locations.

It’s easier to do the interpolation in 2D.

Is there a difference?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

How should we interpolate the information from vertices $p_1$ and $p_2$ at the pixel corresponding to ray $R$?

$z = 0$  $z = 1$
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

1. $R$ intersects the projected line segment in the middle:
   - We should use equal contributions from $p_1$ and $p_2$. 

\[ z = 0 \quad z = 1 \]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

1. \( R \) intersects the projected line segment in the middle:
   - We should use equal contributions from \( p_1 \) and \( p_2 \).

2. \( R \) intersects the 2D line segment closer to \( p_1 \):
   - We should use more information from \( p_1 \) than from \( p_2 \).
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

• How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1 = (x_1, z_1)\) and \(p_2 = (x_2, z_2)\), to find the blending value \(\alpha\) for a pixel falling at position \(x\) in the screen we need to solve:

\[
(1 - \alpha)(x_1, z_1) + \alpha(x_2, z_2) \rightarrow (x, 1)
\]

\[
(1 - \alpha)x_1 + \alpha x_2, (1 - \alpha)z_1 + \alpha z_2 \rightarrow (x, 1)
\]

\[
\frac{(1 - \alpha)x_1 + \alpha x_2}{(1 - \alpha)z_1 + \alpha z_2} = \frac{x}{1}
\]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1 = (x_1, z_1)\) and \(p_2 = (x_2, z_2)\), to find the blending value \(\alpha\) for a pixel falling at position \(x\) in the screen we need to solve:

\[
\frac{(1 - \alpha)x_1 + \alpha x_2}{(1 - \alpha)z_1 + \alpha z_2} = \frac{x}{1}
\]

This is not the same as solving for the blending value in the image plane:

\[
(1 - \alpha) \frac{x_1}{z_1} + \alpha \frac{x_2}{z_2} = \frac{x}{1}
\]

To compute the interpolation weights correctly, we need to perform a perspective divide:

\[
\frac{(1 - \alpha)x_1 + \alpha x_2}{(1 - \alpha)z_1 + \alpha z_2} = \frac{x}{1}
\]
Scan Conversion Example

courtesy of H. Pfister