Shading and Visibility

Michael Kazhdan

(600.457)

HB 13.2 -- 13.8, 14.5
FvDFH 15.4, 15.5, 15.6, 15.7.1, 16.2
3D Rendering Pipeline (for direct illumination)

- **3D Primitives**
  - 3D Modeling Coordinates

- **Modeling Transformation**
  - 3D World Coordinates

- **Camera Transformation**
  - 3D Camera Coordinates

- **Lighting**
  - 3D Camera Coordinates

- **Projection Transformation**
  - 2D Screen Coordinates

- **Clipping**
  - 2D Screen Coordinates

- **Viewport Transformation**
  - 2D Screen Coordinates

- **Scan Conversion**
  - 2D Image Coordinates

- **Image**
  - 2D Image Coordinates

**Diagram:**
- 3D Model
- 2D Screen
- 2D Window
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Scan Conversion

2D Image Coordinates

Image

3D Model

2D Window

2D Screen
Overview

• Scan conversion
  ◦ Figure out which pixels to fill

• Shading
  ◦ Determine a color for each filled pixel

• Depth test
  ◦ Determine when the color of a pixel comes from the front-most primitive
Polygon Shading

• Simplest shading approach is to perform independent lighting calculation for every pixel

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + K_S \langle \hat{V}, \hat{R}_i \rangle^n I_i \right) \]
Polygon Shading

- Can take advantage of spatial coherence
  - Illumination calculations for pixels covered by same primitive are related to each other

\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right) \]
Polygon Shading Algorithms

• Flat Shading
• Gouraud Shading
• Phong Shading
Flat Shading

- Can take advantage of spatial coherence
  - Make the lighting equation constant over the surface of each primitive

<table>
<thead>
<tr>
<th></th>
<th>Surface Normal</th>
<th>Light Direction</th>
<th>View Direction</th>
</tr>
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<tbody>
<tr>
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<td>-</td>
<td>-</td>
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\[
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\[ I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right) \]
Flat Shading

- Illuminate as though all light sources are directional, the polygon is flat, and is viewed from infinitely far away
  - $\langle \vec{N}, \vec{L}_i \rangle$ constant over surface
  - $\langle \vec{V}, \vec{R}_i \rangle$ constant over surface
  - $I_i$ constant over surface

$$I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \vec{N}, \vec{L}_i \rangle I_i + K_S \langle \vec{V}, \vec{R}_i \rangle^n I_i \right)$$
Flat Shading

• One lighting calculation per polygon
  ◦ Assign all pixels inside each polygon the same color
Flat Shading

- Objects look like they are composed of polygons
  - OK for polyhedral objects
  - Not so good for smooth surfaces
Polygon Shading Algorithms

• Flat Shading
• **Gouraud Shading**
• Phong Shading
Gouraud Shading

- What if smooth surface is represented by polygonal mesh with a normal at each vertex?

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + K_S \langle \hat{V}, \hat{R}_i \rangle^n I_i \right)
\]
Gouraud Shading

• One lighting calculation per vertex
  ◦ Assign pixel colors inside polygon by interpolating colors computed at vertices
Gouraud Shading

- Linearly interpolate colors at vertices down and across scan lines
Gouraud Shading

• Linearly interpolate colors at vertices down and across scan lines

\[ A = (1 - \alpha) \cdot I_1 + \alpha \cdot I_2 \]
\[ B = (1 - \beta) \cdot I_2 + \beta \cdot I_3 \]

Note: The values of \( \alpha \) and \( \beta \) only need to be updated as we move to the next scan-line. The value of \( \gamma \) needs to be updated as we advance along the scan-line.
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Continuous shading over adjacent polygons
Gouraud Shading

• Produces smoothly shaded polygonal mesh
  ◦ Continuous shading over adjacent polygons

What happens with large polygon & spotlight?
Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Continuous shading over adjacent polygons

What happens with large polygon & spotlight?
Polygon Shading Algorithms

- Flat Shading
- Gouraud Shading
- Phong Shading
Phong Shading

- One lighting calculation per pixel
  - Approximate surface normals for points inside polygons by linear interpolation of normals from vertices

\[
I = I_E + K_A I_{AL} + \sum_i \left( K_D \langle \hat{N}, \hat{L}_i \rangle I_i + K_S \langle \hat{V}, \hat{R}_i \rangle^n I_i \right)
\]
Phong Shading

- Linearly interpolate surface normals at vertices down and across scan lines

\[
\vec{A} = (1 - \alpha) \cdot \vec{N}_1 + \alpha \cdot \vec{N}_2 \\
\vec{B} = (1 - \beta) \cdot \vec{N}_2 + \beta \cdot \vec{N}_3 \\
\vec{N} = (1 - \gamma) \cdot \vec{A} + \gamma \cdot \vec{B}
\]
Phong Shading

- Linearly interpolate surface normals at vertices down and across scan lines
Polygon Shading Algorithms

- Wireframe
- Flat
- Gouraud
- Phong
Lighting
Camera Transformation
3D Rendering Pipeline (for direct illumination)
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Hidden Surface Removal (HSR)

• Motivation

• Algorithms for HSR
  ○ Back-face detection
  ○ Depth sort
  ○ Ray casting
  ○ $z$-buffer
Motivation

In general, we don’t want to draw surfaces that are not visible to the viewer:

• Surfaces may be back-facing.
• Surfaces may intersect in 3D.
• Surfaces may intersect in the image plane.
Somewhere in here we have to decide which objects are visible, and which objects are hidden.
Visibility algorithms

[Sutherland '74]
Overview

• Motivation

• Algorithms for HSR
  ○ Back-face detection
  ○ BSP-Trees
  ○ Ray casting
  ○ z-buffer
Q: How do we test for back-facing polygons?

A: Dot product of the normal and view directions.

If $\langle \vec{V}, \vec{N} \rangle > 0$, then polygon is back-facing.
Back-face detection

This method breaks down for:

• Overlapping primitives
• Non-solid models and/or models without a well defined orientation.

In general, back-face removal expected to remove $\approx$ half of polygon surfaces from further visibility tests.
A polygon is back-facing if \( \langle \vec{V}, \vec{N} \rangle > 0 \iff \vec{N}_z > 0 \)
A polygon is back-facing if $\langle \vec{V}, \vec{N} \rangle > 0 \iff \vec{N}_z > 0$

Note: When your graphics card does this, it does not use the normals you provide at the vertices for lighting. Instead it uses the cross-product of the triangle vertices, so make sure that the ordering of the vertices is consistent (e.g. CCW)
View-frustrum culling

If the shape is outside the viewing volume, we don’t have to draw it.
View-frustrum culling

If the shape is outside the viewing volume, we don’t have to draw it.
View-frustrum culling

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Image

Trivial Reject
Ideal Solution

Painter’s Algorithm:

• Sort primitives front to back and draw the back ones first, over-writing pixel values with information from the front primitives as they are processed.

Problem:

• In general you can’t sort the primitives.
• ...Unless you are allowed to split them
BSP-Tree Rendering (Object Precision)

- BSP-Trees recursively partition space by planes
  - Given two primitives on either side of a plane, the one on the opposite side from the camera will always be further away.
  - Draw the further side first, and then draw the closer one
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
  • Draw left side of 1
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
    • Draw left side of 3
    • Draw right side of 3
  • Draw left side of 1
BSP-Tree Rendering (Object Precision)

• Draw further half first, then the closer one.
  • Draw right side of 1
    • Draw left side of 3
    • Draw D
  • Draw right side of 3
  • Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
    - Draw right side of 3
      - Draw left side of 5
      - Draw right side of 5
  - Draw left side of 1
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
    - Draw D
  - Draw right side of 3
    - Draw left side of 5
    - Draw E
    - Draw right side of 5
  - Draw left side of 1
BSP-Tree Rendering  (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
    - Draw right side of 3
      - Draw left side of 5
        - Draw E
      - Draw right side of 5
        - Draw F
  - Draw left side of 1
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    - Draw left side of 3
      - Draw D
    - Draw right side of 3
      - Draw left side of 5
        - Draw E
      - Draw right side of 5
        - Draw F
  - Draw left side of 1
    - Draw left side of 2
    - Draw right side of 2
BSP-Tree Rendering (Object Precision)

- Draw further half first, then the closer one.
  - Draw right side of 1
    - Draw left side of 3
      - Draw D
      - Draw right side of 3
        - Draw left side of 5
          - Draw E
          - Draw right side of 5
            - Draw F
  - Draw left side of 1
    - Draw left side of 2
      - Draw left side of 4
      - Draw right side of 4
    - Draw right side of 2
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      - Draw right side of 4
    - Draw right side of 2
BSP-Tree Rendering (Object Precision)

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        - Draw B
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BSP-Tree Rendering (Object Precision)

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    - Draw left side of 2
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      - Draw right side of 4
        - Draw B
    - Draw right side of 2
      - Draw C
3D Rendering Pipeline

Binary Space Partition:
- View Independent
- Linear-time depth sort
Ray Casting

- Fire a ray for every pixel
  - If ray intersects multiple objects, take the closest
Ray Casting Pipeline

Ray casting
- \( P(p \log n) \) for \( p \) pixels
- May (or not) use pixel coherence
- Simple, but generally not used
**z-Buffer**

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer

![Diagram of z-Buffer concept]

- $d=1$
- $d=2$
**z-Buffer**

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer

---

**Case 1 (Blue before Red):**

- Blue $\rightarrow (d = 1) < (d = \infty)$:
  - Set $RGB = (0,0,1), d = 1$
- Red $\rightarrow (d = 2) > (d = 1)$:
  - Don’t change pixel
**z-Buffer**

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**Case 1 (Blue before Red):**
- Blue $\rightarrow (d = 1) < (d = \infty)$:
  - Set $RGB = (0,0,1), d = 1$
- Red $\rightarrow (d = 2) > (d = 1)$:
  - Don’t change pixel

**Case 2 (Red before Blue):**
- Red $\rightarrow (d = 2) > (d = \infty)$:
  - Set $RGB = (1,0,0), d = 2$
- Blue $\rightarrow (d = 1) < (d = 2)$:
  - Set $RGB = (0,0,1), d = 1$
z-Buffer

- Store color & depth of closest object at each pixel
  - Initialize depth of each pixel to $\infty$
  - Update only pixels whose depth is closer than in buffer
  - Depths are interpolated from vertices, just like colors

\[
A = (1 - \alpha) \cdot d_1 + \alpha \cdot d
\]

\[
B = (1 - \beta) \cdot d_2 + \beta \cdot d_3
\]

\[
d = (1 - \gamma) \cdot A + \gamma \cdot B
\]
$z$-Buffer

Edge anti-aliasing becomes difficult because you want multiple triangles to write to the same pixel.

- Who sets the $z$-value?
Alpha values can cause problems:

- Need to know the ordering of primitives behind pixels for $\alpha$-blending
- $\alpha$-buffer supports linked list of surfaces at each pixel for better transparency support
- $\alpha$-buffer also helps with anti-aliasing
3D Rendering Pipeline

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**Image**

**z-buffer comments**
- Polygons rasterized in any order
- Requires additional memory
  - $z$-buffer size $\approx$ frame buffer
- This is what your graphics card does!
3D Rendering Pipeline (for direct illumination)

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2D Window
How do we average information from the three vertices of a triangle?

- Interpolate using weights determined by the 2D screen space projection.
- Interpolate using weights determined by the 3D locations.

It’s easier to do the interpolation in 2D.

Is there a difference?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

How should we interpolate the information from vertices \( p_1 \) and \( p_2 \) at the pixel corresponding to ray \( R \)?
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

1. $R$ intersects the projected line segment in the middle:
   - We should use equal contributions from $p_1$ and $p_2$. 

\[ z = 0 \quad z = 1 \]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

1. $R$ intersects the projected line segment in the middle:
   - We should use equal contributions from $p_1$ and $p_2$.

2. $R$ intersects the 2D line segment closer to $p_1$:
   - We should use more information from $p_1$ than from $p_2$. 

\[ z = 0 \quad z = 1 \]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

• How do we interpolate correctly?

\[ z = 0 \quad z = 1 \]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

- How do we interpolate correctly?

**Recall**: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1 = (x_1, z_1)\) and \(p_2 = (x_2, z_2)\), to find the blending value \(\alpha\) for a pixel falling at position \(x\) in the screen we need to solve:

\[
(1 - \alpha)(x_1, z_1) + \alpha(x_2, z_2) \rightarrow (x, 1)
\]

\[
((1 - \alpha)x_1 + \alpha x_2, (1 - \alpha)z_1 + \alpha z_2) \rightarrow (x, 1)
\]

\[
\frac{(1 - \alpha)x_1 + \alpha x_2}{(1 - \alpha)z_1 + \alpha z_2} = \frac{x}{1}
\]
Scan Conversion Example

A line segment in 2D projected onto a 1D screen.

• How do we interpolate correctly?

Recall: The 2D point \((x, z)\) maps to the point \((x/z)\) in 1D.

If \(p_1 = (x_1, z_1)\) and \(p_2 = (x_2, z_2)\), to find the blending value \(\alpha\) for a pixel falling at position \(x\) in the screen we need to solve:

\[
\frac{(1 - \alpha)x_1 + \alpha x_2}{(1 - \alpha)z_1 + \alpha z_2} = \frac{x}{1}
\]

This is not the same as solving for the blending value in the image plane:

\[
(1 - \alpha) \frac{x_1}{z_1} + \alpha \frac{x_2}{z_2} = \frac{x}{1}
\]

To compute the interpolation weights correctly, we need to perform a perspective divide:
Scan Conversion Example

courtesy of H. Pfister