Radiosity

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(600.357 / 600.457)
Overview

- Ray Tracing Revisited
- Radiosity
Ray Casting

Ray tracing is based on the Phong lighting model:

- A surface reflects light non-uniformly, with stronger reflection in the specular direction:

\[ I = I_E + K_A I_A + \sum_L (K_D \langle N, L \rangle + K_S \langle V, R \rangle^n) I_L S_L \]
Ray Casting

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- Specular Contribution
- Specular Lobe
Ray Tracing

Ray tracing is based on the Phong lighting model:

As a result, when we cast secondary rays, we cast them in the reflected direction to maximize the contribution to the lighting computation.

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Ray Tracing

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This is a good lighting model when the surface material is highly specular. But… not all materials are specular.

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Ray Tracing

Advantage:

- It does a good job of capturing the specular properties of materials.
Ray Tracing

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  ◦ It does a good job of capturing the specular properties of materials.

Disadvantages:
  ◦ Difficult to support soft shadows from area lights
  ◦ Difficult to support caustics
  ◦ Need the ambient term as a hack for the global illumination.
Lighting

What do we really want to compute?
Lighting

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The accumulation of light coming in from all directions, not just the specular one.
Lighting

What do we really want to compute?

The brightness of the light that reaches the camera from some point $p$ in the scene is the sum:

- Of the light emitted from $p$, to the camera plus the sum:
  - Of the light emanating from every point in the scene,
  - Scaled by the extent to which is reflected through $p$. 
Lighting

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What do we really want to compute?

Light arriving at $e$ from $p$

Fraction of light arriving from $s$ that is reflected towards $e$

Light emitted from $p$ to $e$

Amount of light arriving at $p$ from $s$

$$B(p \rightarrow e) = E(p \rightarrow e) + \int_S F_r(s \rightarrow p \rightarrow e)B(s \rightarrow p)ds$$
Lighting

Challenge:

- The integral needs to be computed over a larger number of points because the function is discontinuous.

\[
B(p \to e) = E(p \to e) + \int_S F_r(s \to p \to e)B(s \to p)ds
\]
Lighting

Challenge:

- The integral needs to be computed over a larger number of points because the function is discontinuous.
- The function is recursive since the amount of light leaving a point depends on the amount of light entering it.

\[ B(p \rightarrow e) = E(p \rightarrow e) + \int_{s} F_r(s \rightarrow p \rightarrow e)B(s \rightarrow p)ds \]
Radiosity

Make simplifying assumptions about the scene:

- All lights behave as point lights
  - Since surfaces can emit light, lights can be modeled by emissive surfaces.

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Radiosity

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- All lights behave as point lights
  - Since surfaces can emit light, lights can be modeled by emissive surfaces.
- Objects are Lambertian
  - Perceived brightness is equal in all directions.

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Lambertian Lighting

The perceived brightness is independent of the viewer’s position with respect to the surface.

Under the Lambertian lighting model, point $s$ appears equally bright to $p_1$ and $p_2$. 
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However, a patch of size $A_1$ about $p_1$ gets contributions from a patch of size $A_1 / \cos \theta$ about $s$. 
Lambertian Lighting

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Under the Lambertian lighting model, point \( s \) appears equally bright to \( p_1 \) and \( p_2 \).

However, a patch of size \( A_1 \) about \( p_1 \) gets contributions from a patch of size \( A_1 / \cos \theta \) about \( s \).

Thus, under the Lambertian lighting model, the amount of light \( s \) emits/reflects towards a point \( p \) falls off as \( \cos \theta \).
Radiosity

Make simplifying assumptions about the scene:

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  - Since surfaces can emit light, lights can be modeled by emissive surfaces.
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So how does this affect the lighting equation?
Radiosity

So how does this affect the lighting equation?

- Since emitting surfaces act as point lights, perceived brightness from the point $p$ is constant in all directions.

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\]
Radiosity

So how does this affect the lighting equation?

- Since emitting surfaces act as point lights, perceived brightness from the point $p$ is constant in all directions.

$$B(p \rightarrow e) = E(p) + \int_{s} F_r(s \rightarrow p \rightarrow e) B(s \rightarrow p) ds$$
Radiosity

So how does this affect the lighting equation?

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- Since the surface is Lambertian, the perceived brightness of the light is independent of the viewer’s position.

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- Since the surface is Lambertian, the perceived brightness of the light is independent of the viewer’s position.

\[
B(p) = E(p) + \int_{s} F_{r}(s \rightarrow p) B(s) ds
\]
Radiosity

The fraction of light from point \( s \) that is reflected off of \( p \) is determined by:

- The visibility of \( p \) from \( s \): \( V(s, p) \)
- The angle: \( \theta_{s,p} \)
- The angle: \( \theta_{p,s} \)
- The distance from \( s \) to \( p \): \( \|s - p\| \)
- The material properties at \( p \): \( \rho(p) \)

\[
F_r(s \to p) = \rho(p)V(s, p) \frac{\cos \theta_{s,p} \cdot \cos \theta_{p,s}}{\|s - p\|^2}
\]

\[
B(p) = E(p) + \int_{s \to p} F_r(s \to p) B(s) ds
\]
Radiosity

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\]

The radiosity equation
Radiosity

Explicitly solving the integral is difficult.

\[ B(p) = E(p) + \rho(p) \int_{s} V(s,p) \frac{\cos \theta_{s,p} \cdot \cos \theta_{p,s}}{\|s - p\|^2} B(s) ds \]

The radiosity equation
Radiosity

Explicitly solving the integral is difficult.

Approximate the solution by decomposing surfaces into patches and doing a discrete summation:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

Form Factor
Form Factor

The form factor $F_{j,i}$ is the fraction of the power leaving patch $P_j$ that is received by patch $P_i$:

- $A_i F_{i,j} = A_j F_{j,i}$
- $F_{j,i} \geq 0$
- $\sum_i F_{j,i} \leq 1$
- $F_{ii} = 0$ unless the patch is concave
Radiosity

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Approximate the solution by decomposing surfaces into patches and doing a discrete summation:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} \cdot B_j \]

This amounts to solving a linear system of equations:

- \( E_i, \rho_i, \) and \( F_{j,i} \) are given,
- \( B_i \) are the unknowns.
Radiosity

Re-ordering terms in the equation gives:

\[ B_i = E_i + \rho_i \sum_{j=1}^{n} F_{j,i} B_j \]

\[ E_i = B_i - \rho_i \sum_{j=1}^{n} F_{j,i} B_j \]
Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors

\[
\begin{bmatrix}
1 - \rho_1 F_{1,1} & \cdots & -\rho_1 F_{1,n} \\
-\rho_2 F_{2,1} & 1 - \rho_2 F_{2,2} & \cdots & -\rho_2 F_{2,n} \\
& \ddots & \ddots & \ddots \\
-\rho_{n-1} F_{n-1,1} & \cdots & -\rho_{n-1} F_{n-1,n} \\
-\rho_n F_{n,1} & \cdots & 1 - \rho_n F_{n,n}
\end{bmatrix}
\begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix}
= 
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]
Solving the System of Equations

- Solution methods:
  - **Invert the matrix** \( O(n^3) \)
  - Iterative methods – \( O(n^2) \)
  - Progressive methods – \(< O(n^2)\)

\[
\begin{bmatrix}
1 - \rho_1 F_{1,1} & \cdots & \cdots & -\rho_1 F_{1,n} \\
-\rho_2 F_{2,1} & 1 - \rho_2 F_{2,2} & \cdots & -\rho_2 F_{2,n} \\
\vdots & \vdots & \ddots & \vdots \\
-\rho_{n-1} F_{n-1,1} & \cdots & -\rho_{n-1} F_{n-1,n} \\
-\rho_n F_{n,1} & \cdots & 1 - \rho_n F_{n,n}
\end{bmatrix} \begin{bmatrix}
B_1 \\
B_2 \\
\vdots \\
B_n
\end{bmatrix} = \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_n
\end{bmatrix}
\]

\[A \quad b = e\]
Gauss-Seidel Iteration

Initialization:

◦ For each patch $P_i$, initialize its radiosity to be equal to its emissiveness:

\[ B_i = E_i \]
Gauss-Seidel Iteration

Initialization:
- For each patch $P_i$, initialize its radiosity to be equal to its emissiveness:
  \[ B_i = E_i \]

Iteration:
- At each iteration, update the values of each of the $B_i$ based on the values of all the other $B_j$:
  \[ B_i = E_i + \rho_i \sum_{j \neq i} F_{j,i} \cdot B_j \]
Gauss-Seidel Iteration

- Two interpretations:
  - Iteratively relax rows of linear system
  - Iteratively gather radiosity to elements
Gauss-Seidel Iteration

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- Two interpretations:
  - Iteratively relax rows of linear system
  - Iteratively gather radiosity to elements

Limitation:
Can spend a lot of time gathering radiosity from patches that don’t contribute much.
Progressive Radiosity

- Interpretation:
  - Iteratively shoot “unshot” radiosity from elements
  - Select shooters in order of unshot radiosity
Progressive Radiosity

- Adaptive refinement
Summary

If we could, we would compute the lighting by reflecting secondary rays in all directions to compute the contribution of the entire scene to the brightness of a single point.

• Ray-Tracing:
  ◦ Assume that surfaces are specular so that you only need to bounce in a single (specular) direction.

• Radiosity:
  ◦ Assume that surfaces are Lambertian so that they reflect light in the same way in all directions.