Physically Based Rendering
(600.657)
Materials
Material

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- **Plastic**: combine glossy with diffuse

\[ f_r(Kd, Ks, R) = f_{Lambertian}(Kd) + f_{Torrance-Sparrow}(Ks, F_r, R) \]

with:

- \( Kd \): diffuse color
- \( Ks \): specular/gloss color
- \( F_r \): Fresnel dielectric term
- \( R \): Surface roughness
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In general, the properties are described by textures, allowing material properties to vary spatially.
Textures

We have a bijective map $F : \Omega \rightarrow S$, from a domain $\Omega \subset \mathbb{R}^2$ to the surface $S \subset \mathbb{R}^3$.

We also have a set of textures $T : \Omega$ mapping from the domain $\Omega$ to material parameters (e.g. colors, roughness, etc.)
Textures

For a given point of intersection, \( p \in S \), we evaluate the texture to get the material parameter:

\[
Kd(p) = T_{Kd}(F^{-1}(p))
\]
Texture Maps

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A bump map prescribes the normal offset of the desired surface from the surface modeled by the geometry.

The parameterization of the bumped surface is then:

\[ F^{\text{new}}(x, y) = F(x, y) + B(x, y)n(x, y) \]
Bump Maps

\[ F^{new}(x, y) = F(x, y) + B(x, y)n(x, y) \]

To compute the normal of the new surface, we compute the two tangents:

\[
\frac{\partial F^{new}}{\partial x} = \frac{\partial F}{\partial x} + \frac{\partial B}{\partial x} n + B \frac{\partial n}{\partial x}
\]

\[
\frac{\partial F^{new}}{\partial y} = \frac{\partial F}{\partial y} + \frac{\partial B}{\partial y} n + B \frac{\partial n}{\partial y}
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\]

and take the cross-product:

\[
n^{\text{new}} = \frac{\frac{\partial F^{\text{new}}}{\partial x} \times \frac{\partial F^{\text{new}}}{\partial y}}{\left| \frac{\partial F^{\text{new}}}{\partial x} \times \frac{\partial F^{\text{new}}}{\partial y} \right|}
\]

[Wikipedia]
Bump Maps

$$F^{new}(x, y) = F(x, y) + B(x, y)n(x, y)$$

$$n^{new} = \frac{\partial x}{\partial F^{new}} \times \frac{\partial y}{\partial F^{new}}$$

In practice, it may not be possible to explicitly differentiating the texture (e.g. it may be procedural).

However, we can always approximate it by differencing:

$$\frac{\partial B}{\partial x}(x, y) \approx \frac{B(x + \Delta x, y) - B(x, y)}{\Delta x}$$
**Bump Maps**

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\[ \frac{y}{x} \cdot n \approx \frac{y}{x} \cdot \Delta x \]

How should we choose \( \Delta x \)?

A change by a distance of \( \Delta x \) in the parameter domain should correspond in a shift on the order of a pixel in the image.