Physically Based Rendering (600.657)

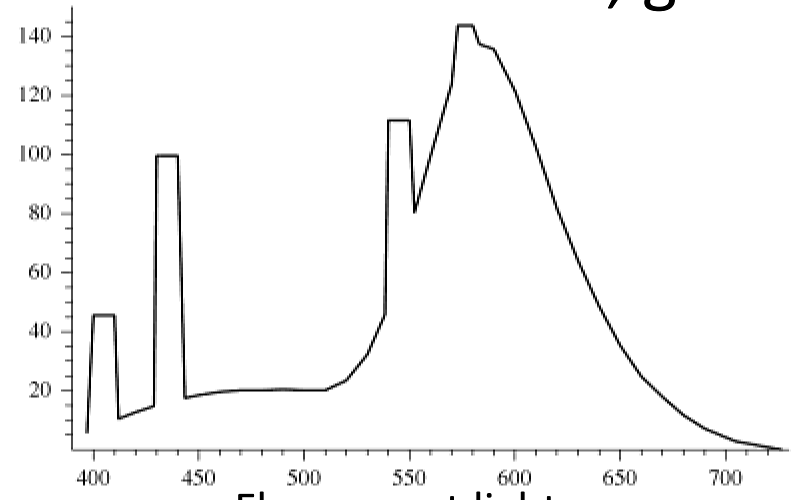
Color & Radiometry

Color

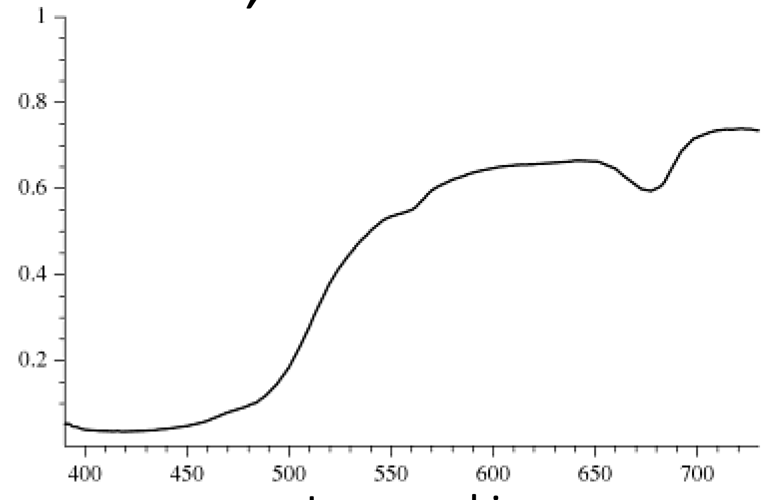
<u>Spectral Power Distribution (SPD)</u>:

Represents the distribution of light across wavelengths.

- Visible light: 370-730 nm
- Blue ~400 nm, green ~550 nm, red ~650 nm



Fluorescent light



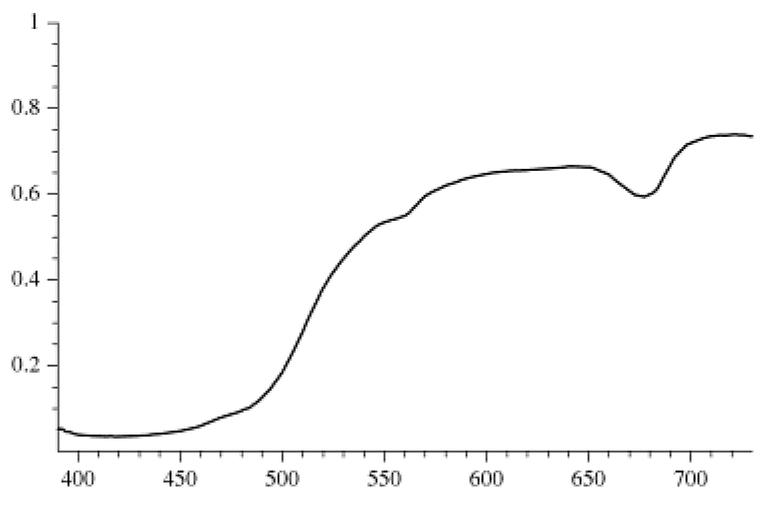
Lemon skin

[Images courtesy of PBRT]

Color

<u>Spectral Power Distribution</u>:

In general, we discretize the spectrum by expressing it as the linear combination of basis functions:

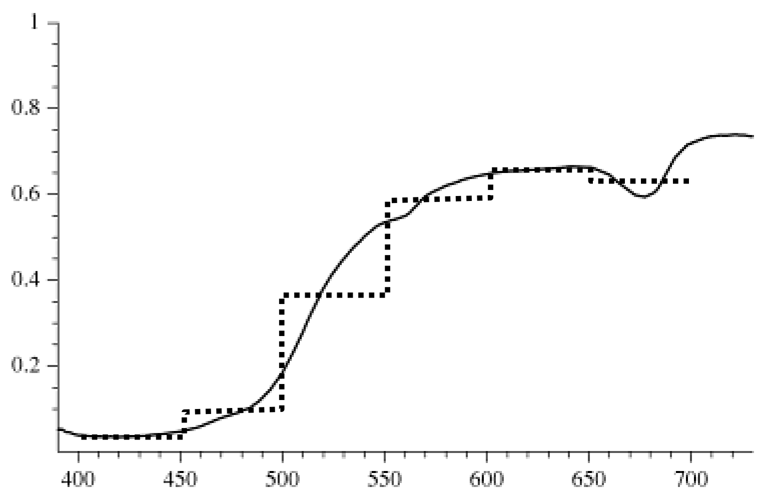


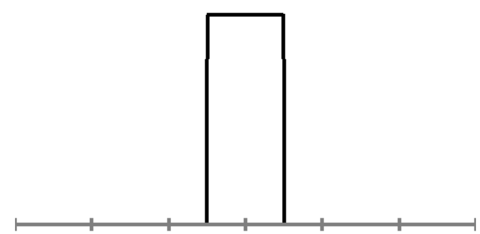
Color

<u>Spectral Power Distribution</u>:

In general, we discretize the spectrum by expressing it as the linear combination of basis functions:

– Piecewise constant



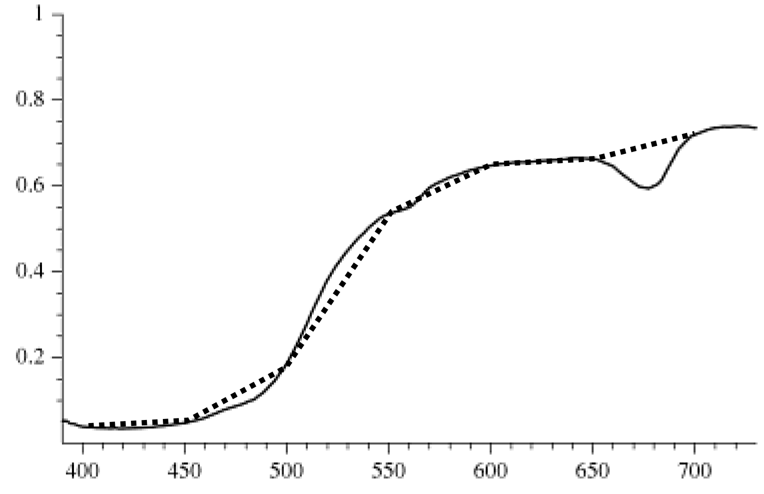


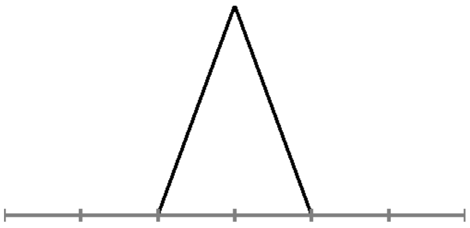
Color

<u>Spectral Power Distribution</u>:

In general, we discretize the spectrum by expressing it as the linear combination of basis functions:

- Piecewise constant
- Piecewise linear



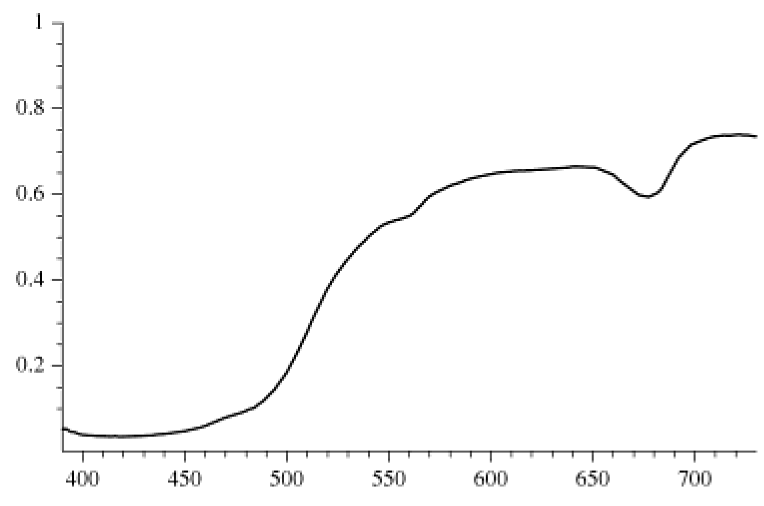


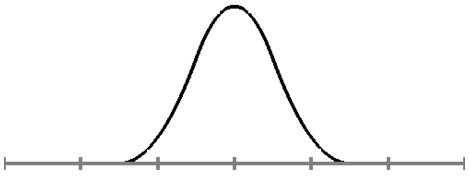
Color

<u>Spectral Power Distribution</u>:

In general, we discretize the spectrum by expressing it as the linear combination of basis functions:

- Piecewise constant
- Piecewise linear
- Etc.





Color

Aside:

Given a set of basis functions $\{B_1(x),\ldots, B_n(x)\}$ how do we find the best approximation to a signal $F(x)$?

Color

<u>Aside</u>:

Given a set of basis functions $\{B_1(x),\ldots, B_n(x)\}$ how do we find the best approximation to a signal $F(x)$?

<u>Goal</u>:

Find coefficients $\{\beta_1,\ldots, \beta_n\}$ minimizing:

$$\int \left\| F(x) - \sum_{i=1}^{n} \beta_i B_i(x) \right\|^2 dx$$

Color

Goal:

Find coefficients $\{\beta_1,\ldots,\beta_n\}$ minimizing:

$$\int\left\|F(x)-\sum_{i=1}^{n}\beta_i B_i(x)\right\|^2 dx = \int\|F(x)\|^2 - 2\sum_{i=1}^{n}\beta_i\int B_i(x)F(x)dx + \sum_{i,j=1}^{n}\beta_i\beta_j\int B_i(x)B_j(x)dx$$

Color

Goal:

Find coefficients $\{\beta_1, \ldots, \beta_n\}$ minimizing:

$$\int \left\| F(x) - \sum_{i=1}^{n} \beta_i B_i(x) \right\|^2 dx = \int \left\| F(x) \right\|^2 - 2\sum_{i=1}^{n} \beta_i \int B_i(x) F(x) dx + \sum_{i,j=1}^{n} \beta_i \beta_j \int B_i(x) B_j(x) dx$$

Taking derivatives with respect to the β_i gives:

$$\sum_{j=1}^{n} \beta_j \int B_i(x) B_j(x) dx = \int B_i(x) F(x) dx$$

Color

Goal:

Find coefficients $\{\beta_1,\ldots,\beta_n\}$ satisfying:

$$\sum_{j=1}^{n}\beta_j\int B_i(x)B_j(x)dx=\int B_i(x)F(x)dx$$

So the coefficients are the solution to the linear system $A\{\beta_i\}=b$ where:

$$A_{ij}=\int B_i(x)B_j(x)dx$$

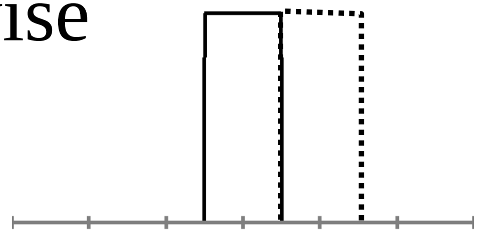
$$b_i=\int B_i(x)F(x)dx$$

Color

Note:

For piecewise constant approximation, the basis functions are orthonormal:

$$A_{ij} = \int B_i(x) B_j(x) dx = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$



Color

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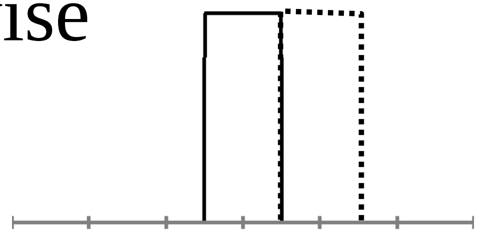


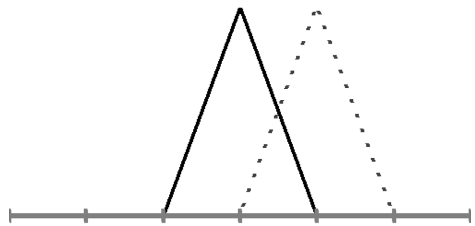
So *A* is the identity and we have:

$$\beta_i = \int_{i-0.5}^{i+0.5} F(x) dx$$

Color

<u>Note</u>:

For piecewise linear approximation, the basis functions are not orthonormal and computing the coefficients requires solving a linear system.

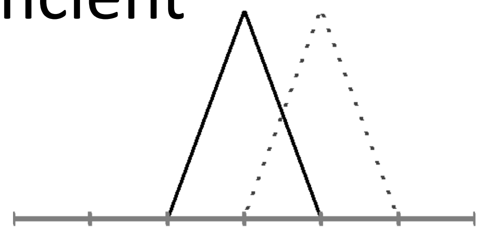


Color

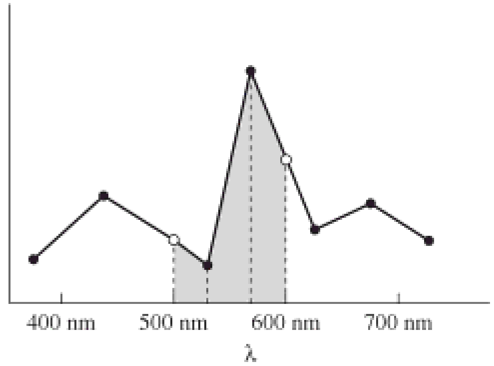
Note:

For piecewise linear approximation, the basis functions are not orthonormal and computing the coefficients requires solving a linear system.

In practice, we still approximate the coefficient by averaging:



$$\beta_i = \int_{i-0.5}^{i+0.5} F(x)dx$$



XYZ Color

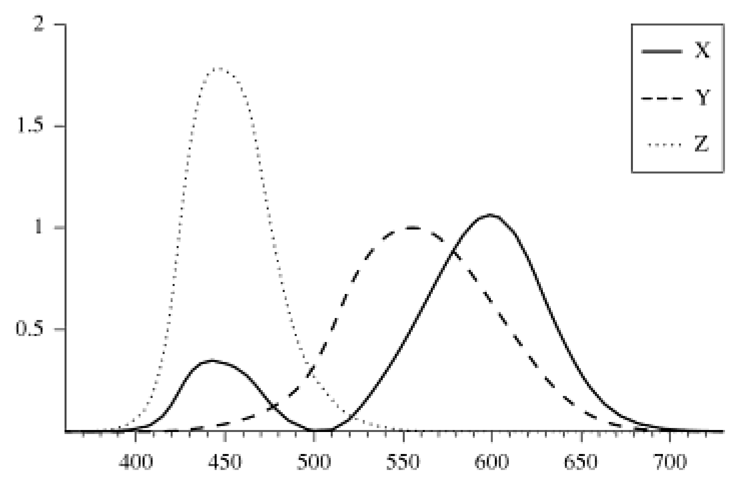
<u>Tristimulus Theory</u>:

For human observers, every SPD can be characterized by three values.
These are obtained by integrating the SPD against *spectral matching curves* $X(\lambda)$, $Y(\lambda)$, $Z(\lambda)$:

$$x_S = \frac{\int S(\lambda)X(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$

$$y_S = \frac{\int S(\lambda)Y(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$

$$z_S = \frac{\int S(\lambda)Z(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$



XYZ Color

Tristimulus Theory:

For human observers, every SPD can be characterized by three values.

These are obtained by integrating the SPD

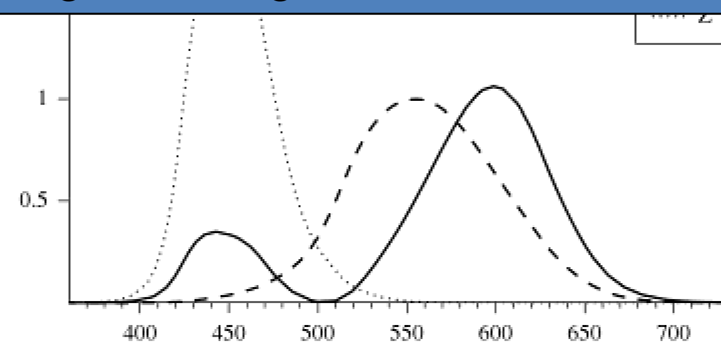
Note:

This does not mean that:

$$S(\lambda)=x_S X(\lambda)+y_S Y(\lambda)+z_S Z(\lambda)$$

$$y_\lambda = \frac{\int S(\lambda)Y(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$

$$z_\lambda = \frac{\int S(\lambda)Z(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$



XYZ Color

<u>Tristimulus Theory</u>:

For human observers, every SPD can be characterized by three values.
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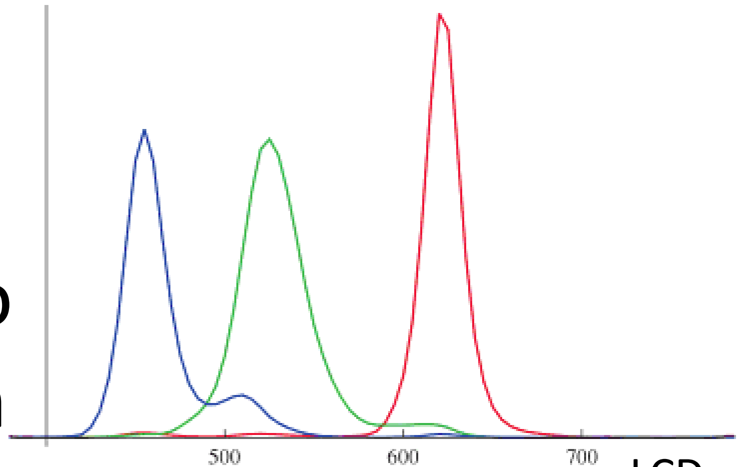
<u>Note</u>:
This does not mean that:

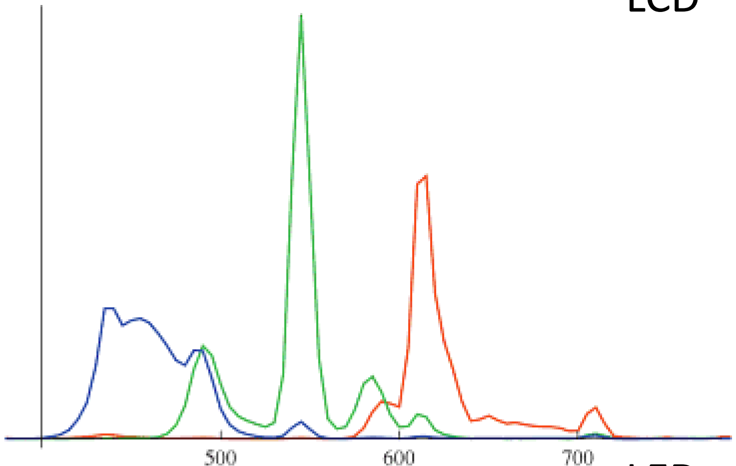
<u>Note</u>:
Many different SPDs can have the same characteristic (x_S,y_S,z_S). These look the same to the human eye and are called <u>metamers</u>.

RGB Color

<u>Displays</u>:

Given a display emitting in red, green, and blue and given an SPD $S(\lambda)$, we want to compute the RGB values such that when the display emits with those values, the color will appear the same as the color represented by $S(\lambda)$.





RGB Color

<u>Displays</u>:

If the red, green, and blue SPDs emission of the display are given by $R(\lambda)$, $G(\lambda)$, $B(\lambda)$, this gives:

$$x_S = \frac{\int S(\lambda)X(\lambda)d\lambda}{\int Y(\lambda)d\lambda} = \frac{\int \big(rR(\lambda) + gG(\lambda) + bB(\lambda)\big)X(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$

$$y_S = \frac{\int S(\lambda)Y(\lambda)d\lambda}{\int Y(\lambda)d\lambda} = \frac{\int \big(rR(\lambda) + gG(\lambda) + bB(\lambda)\big)Y(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$

$$z_S = \frac{\int S(\lambda)Z(\lambda)d\lambda}{\int Y(\lambda)d\lambda} = \frac{\int \big(rR(\lambda) + gG(\lambda) + bB(\lambda)\big)Z(\lambda)d\lambda}{\int Y(\lambda)d\lambda}$$

RGB Color

Displays:

If the red, green, and blue SPDs emission of the display are given by $R(\lambda)$, $G(\lambda)$, $B(\lambda)$, this gives:

$$\begin{pmatrix} x_S \\ y_S \\ z_S \end{pmatrix} = \frac{1}{\int Y(\lambda)d\lambda} \begin{pmatrix} \int R(\lambda)X(\lambda)d\lambda & \int G(\lambda)X(\lambda)d\lambda & \int B(\lambda)X(\lambda)d\lambda \\ \int R(\lambda)Y(\lambda)d\lambda & \int G(\lambda)Y(\lambda)d\lambda & \int B(\lambda)Y(\lambda)d\lambda \\ \int R(\lambda)Z(\lambda)d\lambda & \int G(\lambda)Z(\lambda)d\lambda & \int B(\lambda)Z(\lambda)d\lambda \end{pmatrix} \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

[This is different from the formulation in PBRT!?!]

RGB Color to SPD

Absent specific emission profiles for a particular display, we would like a reasonable way to transform RGB values to SPDs:

- R=G=B $\Rightarrow$ the SPD should be constant
- The SPD should be smooth

RGB Color to SPD

Approach:

Specify SPDs for white, RGB, and CMY. (These can be different for reflection and illumination.)

RGB Color to SPD

Approach:

Specify SPDs for white, RGB, and CMY. (These can be different for reflection and illumination.)

Given an RGB triplet (*r*,*g*,*b*) iteratively subtract off values from the triplet to zero out components and update the SPD accordingly.

RGB Color to SPD

Example $(r_0, g_0, b_0) = (\ 0.6\ ,\ 0.2\ ,\ 0.5\ )$:

1. Green is smallest of three:

$(r_1, g_1, b_1) = (r_0, g_0, b_0) - (g_0, g_0, g_0)$ $\qquad S(\lambda) \mathrel{+}= g_0\, W(\lambda)$

RGB Color to SPD

<u>Example (r_1,g_1,b_1)= (0.4 , 0.0 , 0.3)</u>:

1. Green is smallest of three :

   $(r_1,g_1,b_1) = (r_0,g_0,b_0) - (g_0,g_0,g_0)$ $\quad\quad S(\lambda) \mathrel{+}= g_0\, W(\lambda)$

2. Blue is smallest of two:

   $(r_2,g_2,b_2) = (r_1,g_1,b_1) - (b_1,0,b_1)$ $\quad\quad S(\lambda) \mathrel{+}= b_1\, M(\lambda)$

RGB Color to SPD

Example $(r_2, g_2, b_2) = (\ 0.1\ ,\ 0.0\ ,\ 0.0\ )$:

1. Green is smallest of three :

   $(r_1, g_1, b_1) = (r_0, g_0, b_0) - (g_0, g_0, g_0)$ $\qquad S(\lambda) \mathrel{+}= g_0\, W(\lambda)$

2. Blue is smallest of two:

   $(r_2, g_2, b_2) = (r_1, g_1, b_1) - (b_1, 0, b_1)$ $\qquad S(\lambda) \mathrel{+}= b_1\, M(\lambda)$

3. Red is left:

   $(r_3, g_3, b_3) = (r_2, g_2, b_2) - (r_2, 0, 0)$ $\qquad S(\lambda) \mathrel{+}= r_2\, R(\lambda)$

RGB Color to SPD

Example (r_3,g_3,b_3)= (0.0 , 0.0 , 0.0):

1. Green is smallest of three :

   $(r_1,g_1,b_1) = (r_0,g_0,b_0) - (g_0,g_0,g_0)$ $\qquad S(\lambda) \mathrel{+}= g_0\, W(\lambda)$

2. Blue is smallest of two:

   $(r_2,g_2,b_2) = (r_1,g_1,b_1) - (b_1,0,b_1)$ $\qquad S(\lambda) \mathrel{+}= b_1\, M(\lambda)$

3. Red is left:

   $(r_3,g_3,b_3) = (r_2,g_2,b_2) - (r_2,0,0)$ $\qquad S(\lambda) \mathrel{+}= r_2\, R(\lambda)$

Note:
This mapping from RGB-space to SPDs is not linear.

Radiometry

Terms:

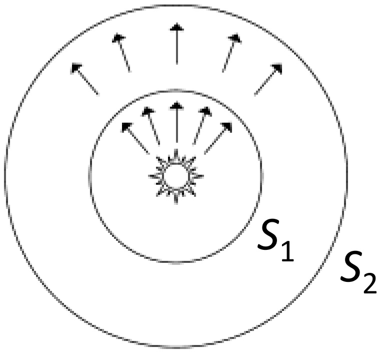
- Flux
- Irradiance / Radiant Exitance
- Intensity
- Radiance

Radiometry

<u>Flux (Φ)</u>:

Measures the total amount of energy passing through a surface per unit time [measured in W].

- Used to describe the total emission from lights.
- (In 2D, the flux through a closed curve around a light source is related to the winding number of the curve.)



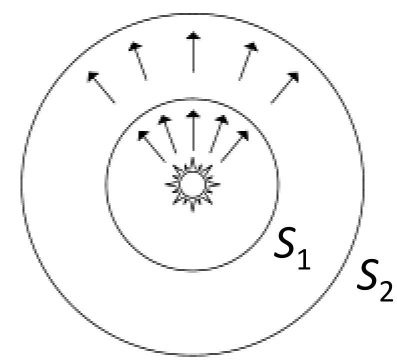
Radiometry

<u>Irradiance / Radiant Exitance (*E*/*M*)</u>:

The area density of flux arriving/leaving a surface [measured in W/m^2].

– Integrating the irradiance / radiant exitance we get the flux:

$$\Phi(S) = \int_{p \in S} E(p)\,dp$$



Radiometry

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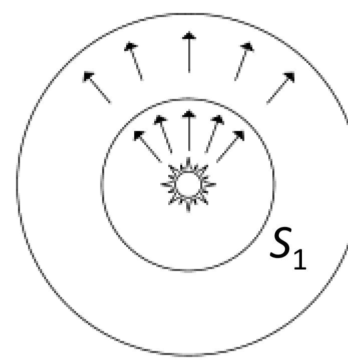
– Integrating the irradiance / radiant exitance we get the flux:

$$\Phi(S) = \int_{p \in S} E(p)\,dp$$

<u>Example</u>:

If a point emits isotropicaly, the radiance on a sphere of radius *r* about the light is:



$$\Phi = \int_{p \in S} E(p)\,dp = E \int_{p \in S} dp = 4\pi r^2 E \quad \Rightarrow \quad E = \frac{\Phi}{4\pi r^2}$$

Radiometry

Irradiance / Radiant Exitance (*E*/*M*):

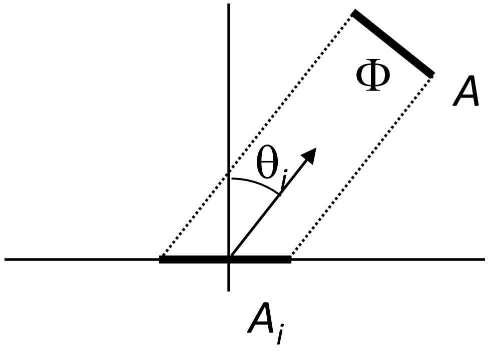
The area density of flux arriving/leaving a surface [measured in W/m²].

– Integrating the irradiance / radiant exitance we get the flux:

$$\Phi(S) = \int_{p \in S} E(p)\,dp$$

Lambert's Law:

Irradiance at a surface varies with the cosine of the angle of incidence:



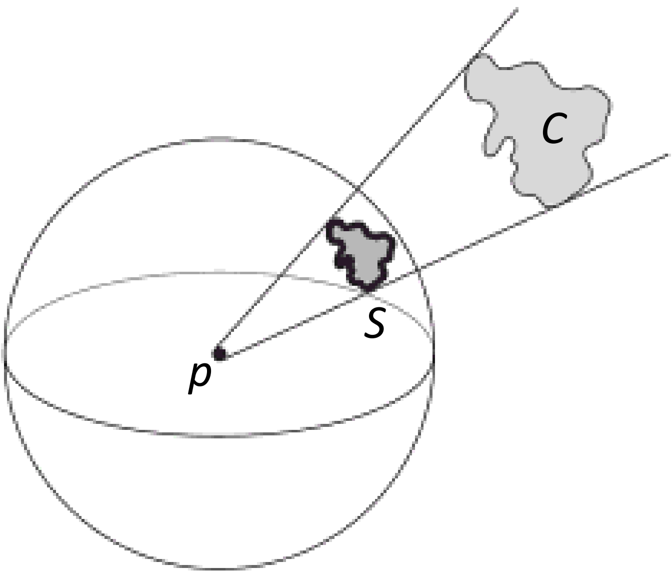
$$E = \frac{\Phi}{A_i} = \frac{\Phi}{A / \cos\theta_i} = \frac{\Phi \cos\theta_i}{A}$$

Radiometry

<u>Intensity (*I*)</u>:

The solid angle density of flux emanating from a point light source [measured in W/steridian].

$$\Phi(C) = \int_S I(\omega) d\omega$$



Radiometry

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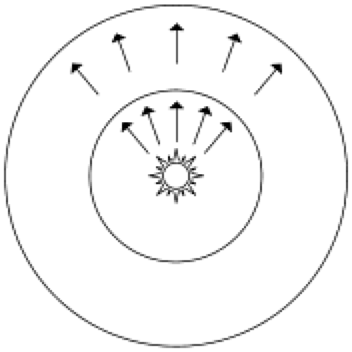
$$\Phi(C) = \int_S I(\omega)d\omega$$

<u>Example</u>:

If a point emits isotropicaly, the intensity at any direction is:

$$\Phi = I \int_{\omega \in S^2} d\omega = 4\pi I \quad \Rightarrow \quad I = \frac{\Phi}{4\pi}$$



Radiometry

Intensity (*I*):

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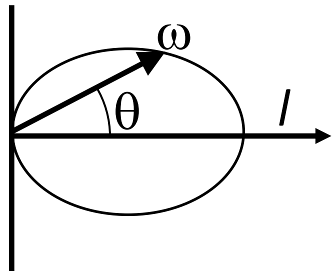
Warn's Spotlight:



$$I(\omega) = \cos^s \theta = \langle \omega, l \rangle^s$$

[Images courtesy of Pat Hanarhan]

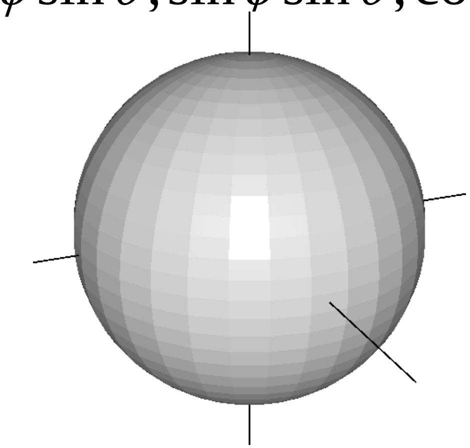
Radiometry

<u>Intensity (*I*)</u>:

The solid angle density of flux emanating from a point light source [measured in W/steridian].

$$\Phi(C) = \int_S I(\omega)\,d\omega$$

$$\left(\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta\right)$$



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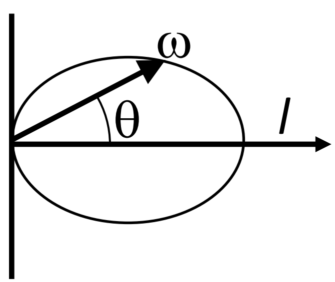


$$I(\omega) = \cos^s\theta = \left\langle \omega, l \right\rangle^s$$

$$\Phi = \int_{HS^2} \left\langle \omega, l \right\rangle^s d\omega = \int_0^{2\pi}\int_0^{\pi/2} \cos^s\theta\sin\theta\,d\theta\,d\phi = 2\pi\int_0^{\pi/2}\cos^s\theta\,d\sin\theta = 2\pi\int_0^1 x^s dx = \frac{2\pi}{s+1}$$

[Images courtesy of Pat Hanarhan]

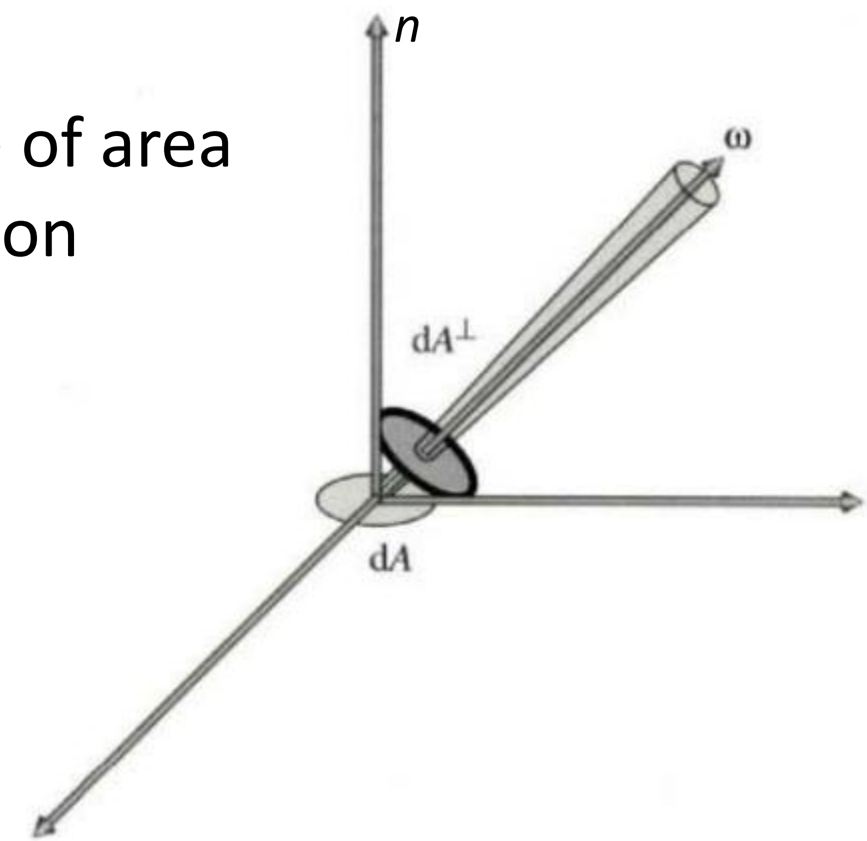
Radiometry

Radiance (L):

The solid angle and area density of flux [measured in W/(steridian·m^2)].

– Note that the measure of area is along $A^{\perp}$, the direction perpendicular to ω.

$$dA = dA^{\perp}\langle n, \omega \rangle$$



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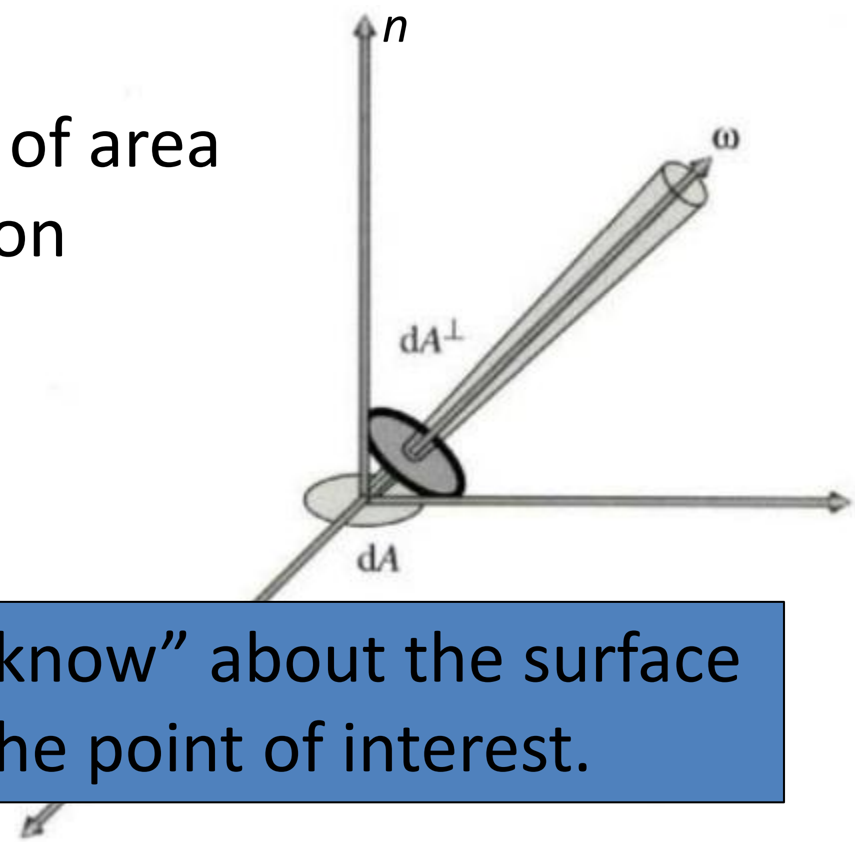


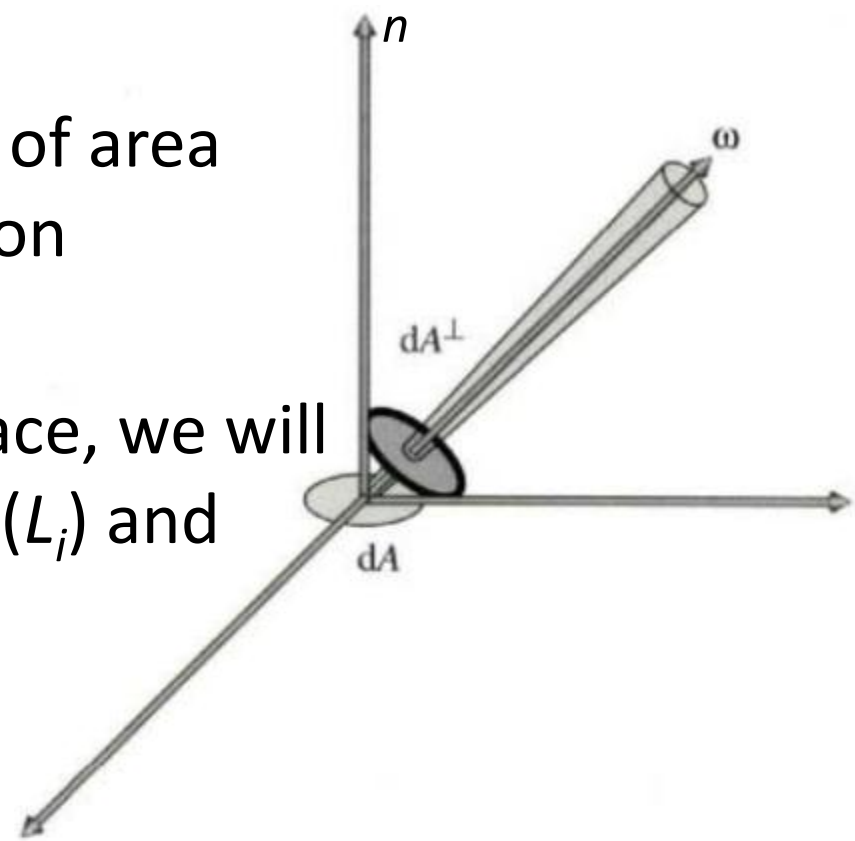
Radiance does not "know" about the surface orientation at the point of interest.

Radiometry

<u>Radiance (*L*)</u>:

The solid angle and area density of flux [measured in W/(steridian·m^2)].

- Note that the measure of area is along $A^{\perp}$, the direction perpendicular to ω.
- For a point on the surface, we will consider both incident (L_i) and exitant (L_o) radiance.



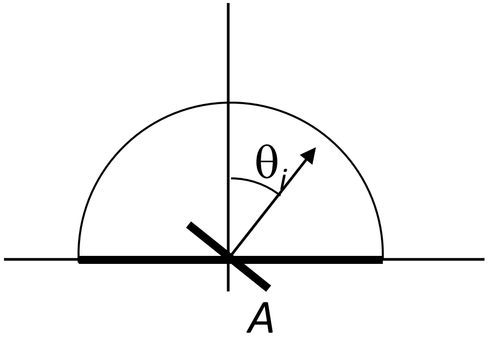
Radiometry

<u>Radiance (*L*)</u>:

The solid angle and area density of flux [measured in W/(steridian·m^2)].

For a patch on a surface, the irradiance is the integral of the radiance over all incoming directions, scaled by the change in area term:

$$E = \int_{\omega \in HS^2} L_i(\omega) \cos\theta \, d\omega$$



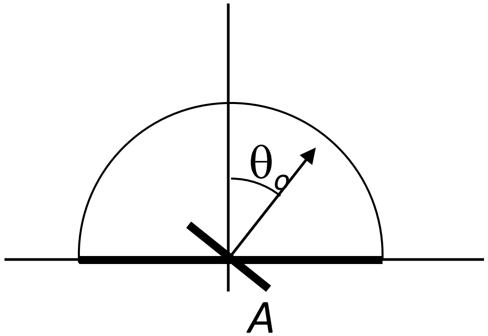
Radiometry

<u>Radiance (*L*)</u>:

The solid angle and area density of flux [measured in W/(steridian·m^2)].

Similarly, the flux emitted from an object is the integral of the radiance in all outgoing directions, scaled by the change in area, and integrated over the surface of the object:

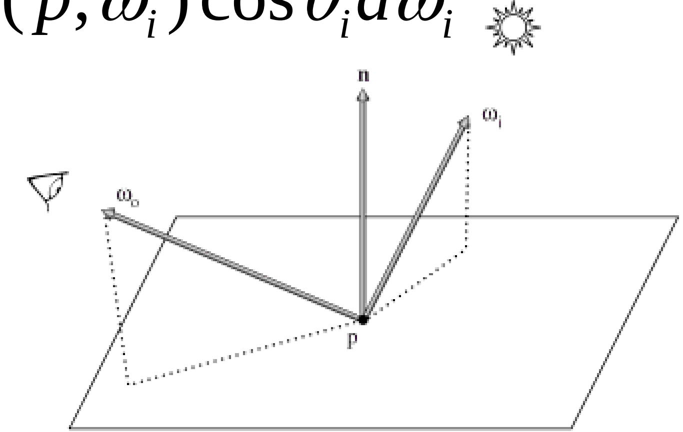
$$\Phi = \int_A \int_{\omega \in HS^2} L_o(\omega) \cos\theta \, d\omega \, dA$$



Bidirectional Reflectance Distribution

Gives the ratio of outgoing radiance in direction ω_o to the irradiance on the surface from incoming direction ω_i.

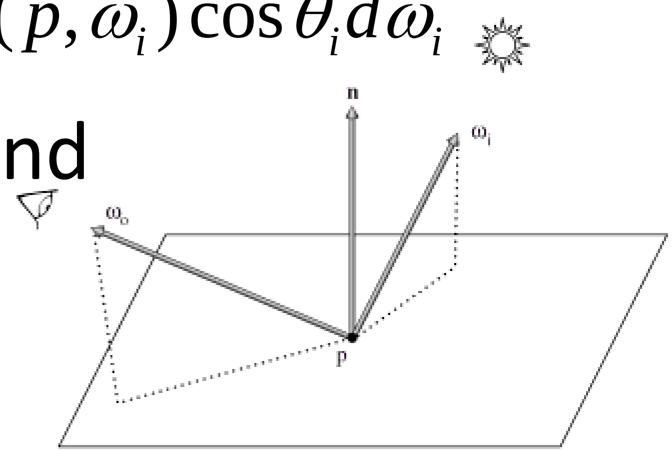
$$f_r(p,\omega_i,\omega_o) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i)\cos\theta_i d\omega_i}$$



BRDF

Gives the ratio of outgoing radiance in direction ω_o to the irradiance on the surface from incoming direction ω_i.

$$f_r(p,\omega_i,\omega_o) = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)} = \frac{dL_o(p,\omega_o)}{L_i(p,\omega_i)\cos\theta_i d\omega_i}$$



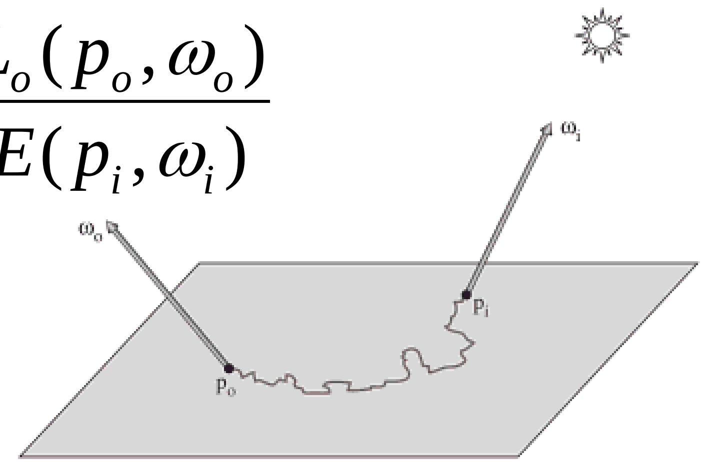
Thus, given a BRDF $f_r(p,\omega_i,\omega_o)$ and the incoming radiance $L_i(p,\omega_i)$, the outgoing radiance in ω_o is:

$$L_o(p,\omega_o) = \int_{S^2} f_r(p,\omega_i,\omega_o) L_i(p,\omega_i)\cos\theta_i d\omega_i$$

Bidirectional Scattering-Surface Reflectance Distribution Function

Gives the ratio of outgoing radiance in direction ω_o at point p_o, to the irradiance on the surface from incoming direction ω_i at point p_i.

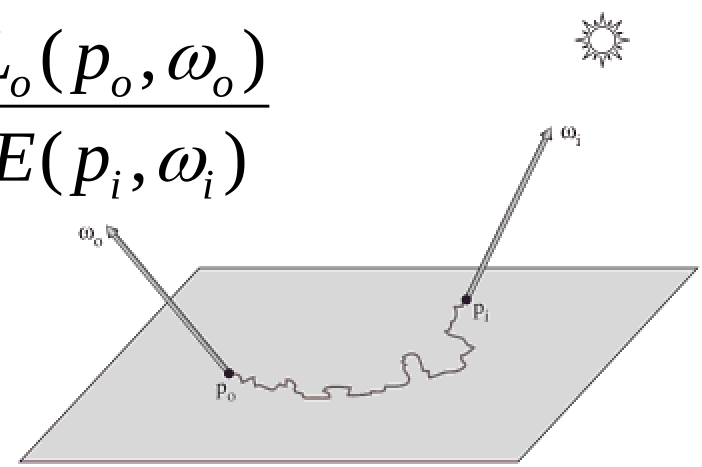
$$S(p_i, \omega_i, p_o, \omega_o) = \frac{dL_o(p_o, \omega_o)}{dE(p_i, \omega_i)}$$



BSSRDF

Gives the ratio of outgoing radiance in direction ω_o at point p_o, to the irradiance on the surface from incoming direction ω_i at point p_i.

$$S(p_i, \omega_i, p_o, \omega_o) = \frac{dL_o(p_o, \omega_o)}{dE(p_i, \omega_i)}$$



Thus, given a BSSRDF *S* and the incoming radiance L_i the outgoing radiance in ω_o at p_o is:

$$L_o(p_o, \omega_o) = \int_A \int_{S^2} S(p_i, \omega_i, p_o, \omega_o) L_i(p_i, \omega_i) \cos\theta_i \, d\omega_i \, dA$$