Large Mesh Deformation Using the Volumetric Graph Laplacian

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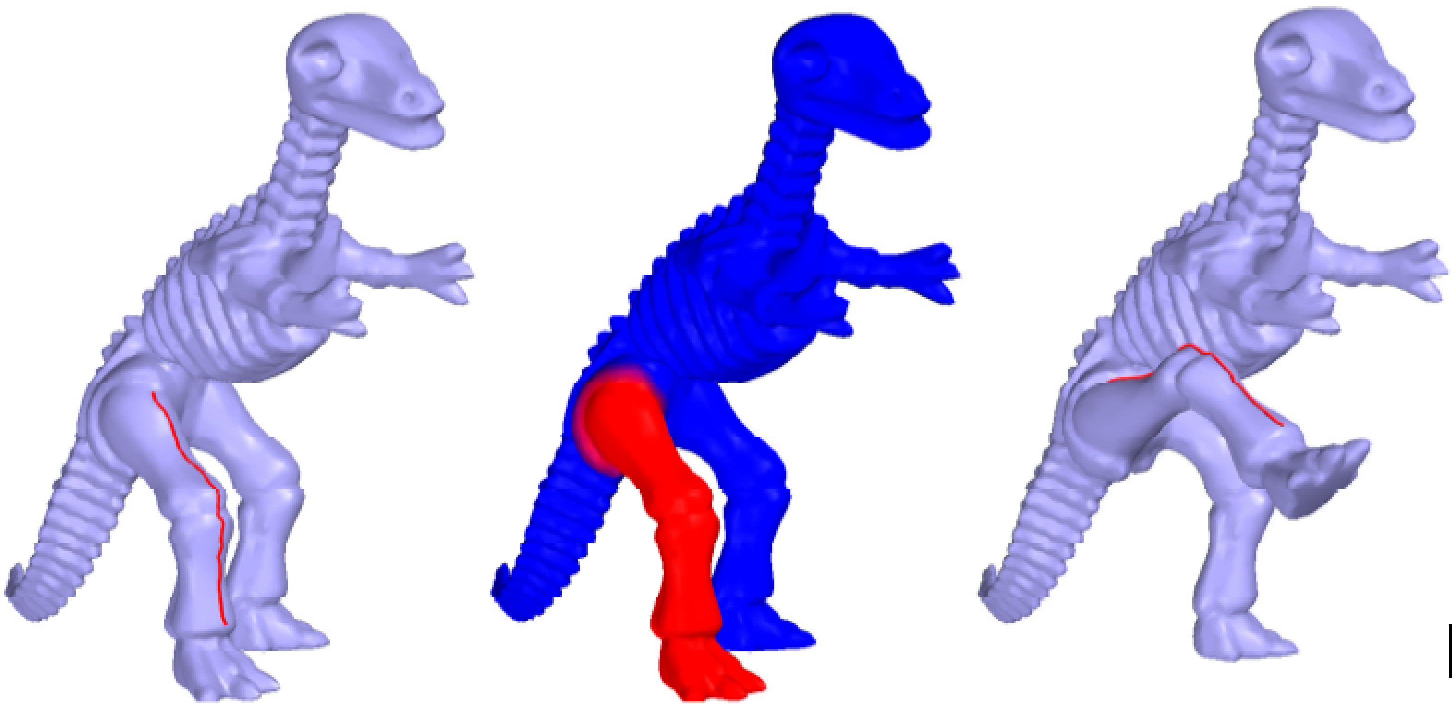

Presented by
Bhaskar Kishore

Outline

- Introduction
- Related Work
- Deformation on Volumetric Graphs
- Deformation from 2D curves
- Results
- Conclusions

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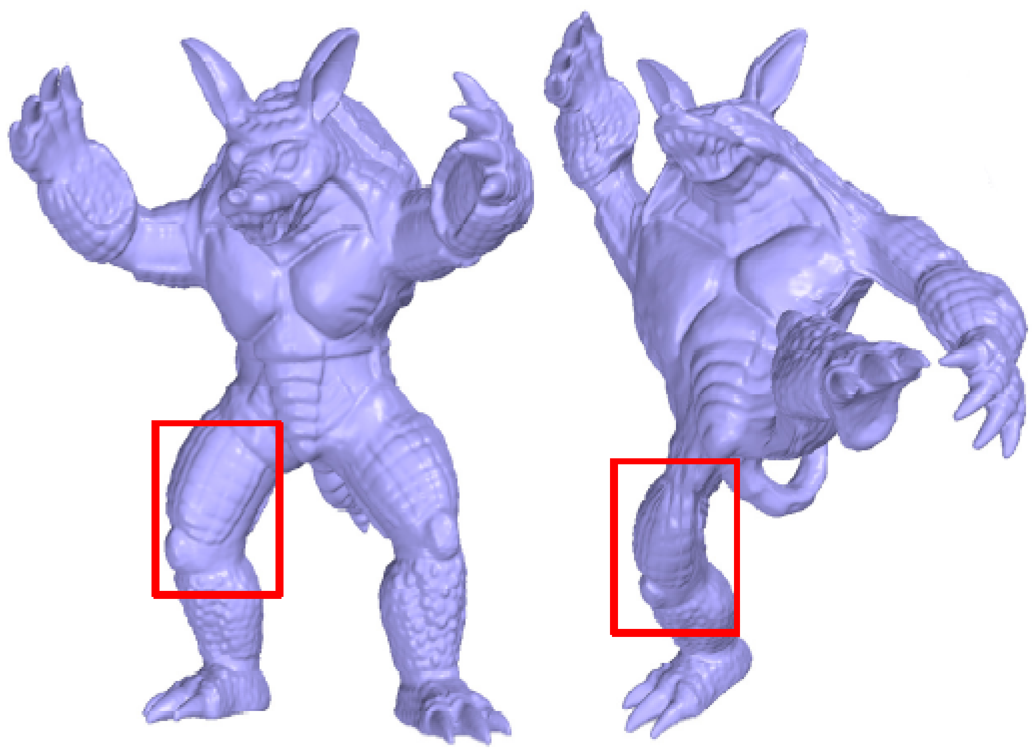
Introduction



- Large deformations are challenging
- Existing techniques often produce implausible results
- Observation
  - Unnatural volume changes
  - Local Self Intersection

Introduction

- Volumetric Graph Laplacian
  - Represent volumetric details as difference between each point in a 3D volume and the average of its neighboring points in a graph.
  - Produces visually pleasing deformation results
  - Preserves surface details
- VGL can impose volume constraints
- Volumetric constraints are represented by a quadric energy function

Introduction

- Volumetric Graph Laplacian
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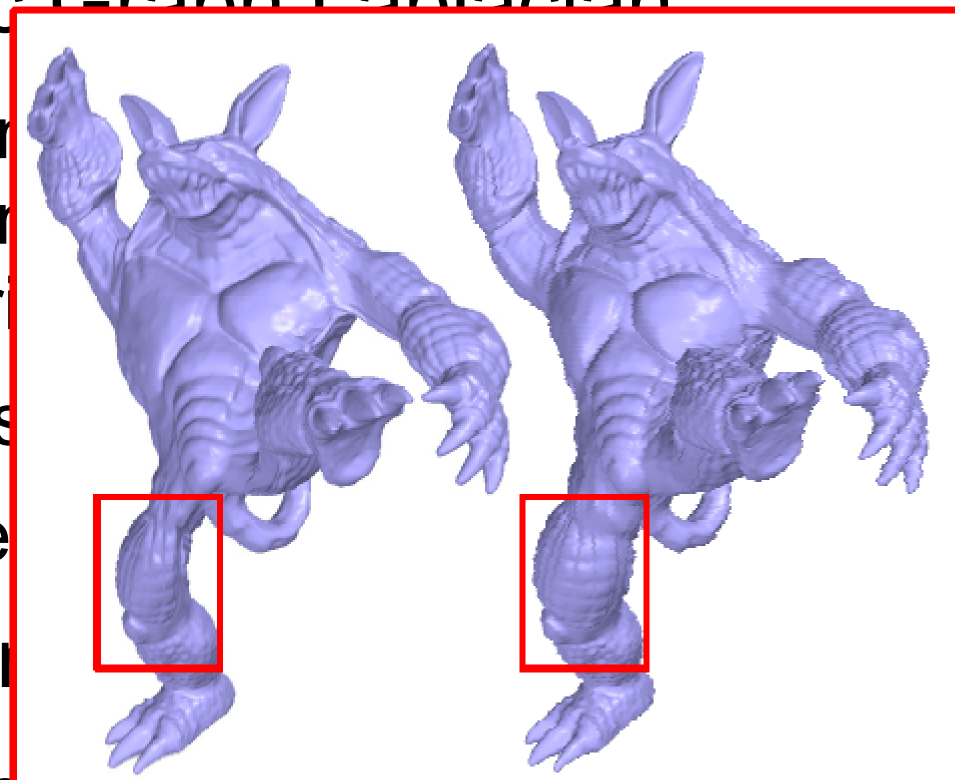
  - Produces n results
  - Preserve
- VGL can i s



- Volumetric constraints are represented by a quadric energy function

Introduction

- To apply VGL to a triangle mesh
  - Construct a volumetric graph which includes
    - Points on the original mesh
    - Points derived from a simple lattice lying inside the mesh
  - Points are connected by graph edges which are a superset of the edges of the original mesh
- Whats nice is that there is no need for volumetric tessellation.
- Deformations are specified by identifying a limited set of points – say a curve

Introduction

- This curve can then be deformed to specify destination
- A quadric energy function is generated
  - Minimum maps the points to their specified destination
  - While preserving surface detail and roughly volume too

Introduction

- Contribution
  - Demonstrate that problem of large deformation can be effectively solved by volumetric differential operator
    - Surface operators can be extended to solids by defining them on tetrahedral mesh
    - But that is difficult, constructing the tetrahedral mesh is hard
    - Existing packages remesh geometry and change connectivity
  - That a volumetric operator can be applied to the easy to build Volumetric graph without meshing int.

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Related Work

- Freeform modeling [Botsch Kobbelt 2004]
- Curve based FFD [Sing and Fiume 1998]
- Lattice based FFD [Sederberg and Parry 1986]
- Displacement volumes [Botsch and Kobbelt 2003]
- Poisson meshes [Yu et al 2004]

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Deformation on Volumetric Graphs

- Let M = (V, K) be a triangular mesh
  - V = $\{p_i \in R^3 \mid 1 \leq i \leq n\}$, is a set of n point position
  - K is a abstract simplicial complex containing three types of elements
    - Vertices {i}
    - Edges {i,j}
    - Faces {i,j,k}

Laplacian Deformation on Abstract Graphs

- Suppose G = (P,E) is a graph
  - P $\{p_i \in R^3 \mid 1 \leq i \leq N\}$, is a set of N point positions
  - E = $\{(i,j) \mid p_i$ is connected to $p_j\}$
- Then Laplacian coordinate δi of a point p_i

$$\delta_i = \mathcal{L}_G(p_i) = p_i - \sum_{j \in \mathcal{N}(i)} w_{ij} p_j, \qquad (1)$$

where N $(i) = \{ j \mid \{i, j\} \in E\}$

- L_G is called the Laplace operator on graph G

Laplacian Deformation on Abstract Graphs

- To control the deformation
  - User inputs deformed positions q_i, $i \in \{1, \ldots, m\}$ for a subset of the N mesh vertices
  - Compute a new (deformed) laplacian coordinate δ'_i for each point *i* in the graph
  - Deformed positions of the mesh vertices p'_i is obtained by solving

$$\min_{p'_i} \left(\sum_{i=1}^{N} \|\mathscr{L}_G(p'_i) - \delta'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2 \right). \qquad (2)$$

Laplacian Deformation on Abstract Graphs

$$\min_{p'_i} \left(\sum_{i=1}^{N} \|\mathscr{L}_G(p'_i) - \delta'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2 \right). \qquad (2)$$

- The first term represents preservation of local detail
- The second term constrains the position of those vertices directly specified by the user
- Alpha is used to balance these two objectives

Laplacian Deformation on Abstract Graphs

- Deformed Laplacian coordinates are computed via

$$\delta'_i = T_i \delta_i$$

- δ_i is the Laplacian in rest pose
- δ'_i is the Laplacian in the deformed pose
- T_i is restricted to rotation and isotropic scale
- Local transforms are propagated from the deformed region to the entire mesh

Constructing a Volumetric Graph

- Build two graphs, G_{in} and G_{out}
- G_{in} prevents large volume changes
- G_{out} prevents local self-intersection
- G_{in} can obtained by tetrahedralizing the interior
  - Difficult to implement
  - Computationally expensive
  - Produces poorly shaped tetrahedra for complex models

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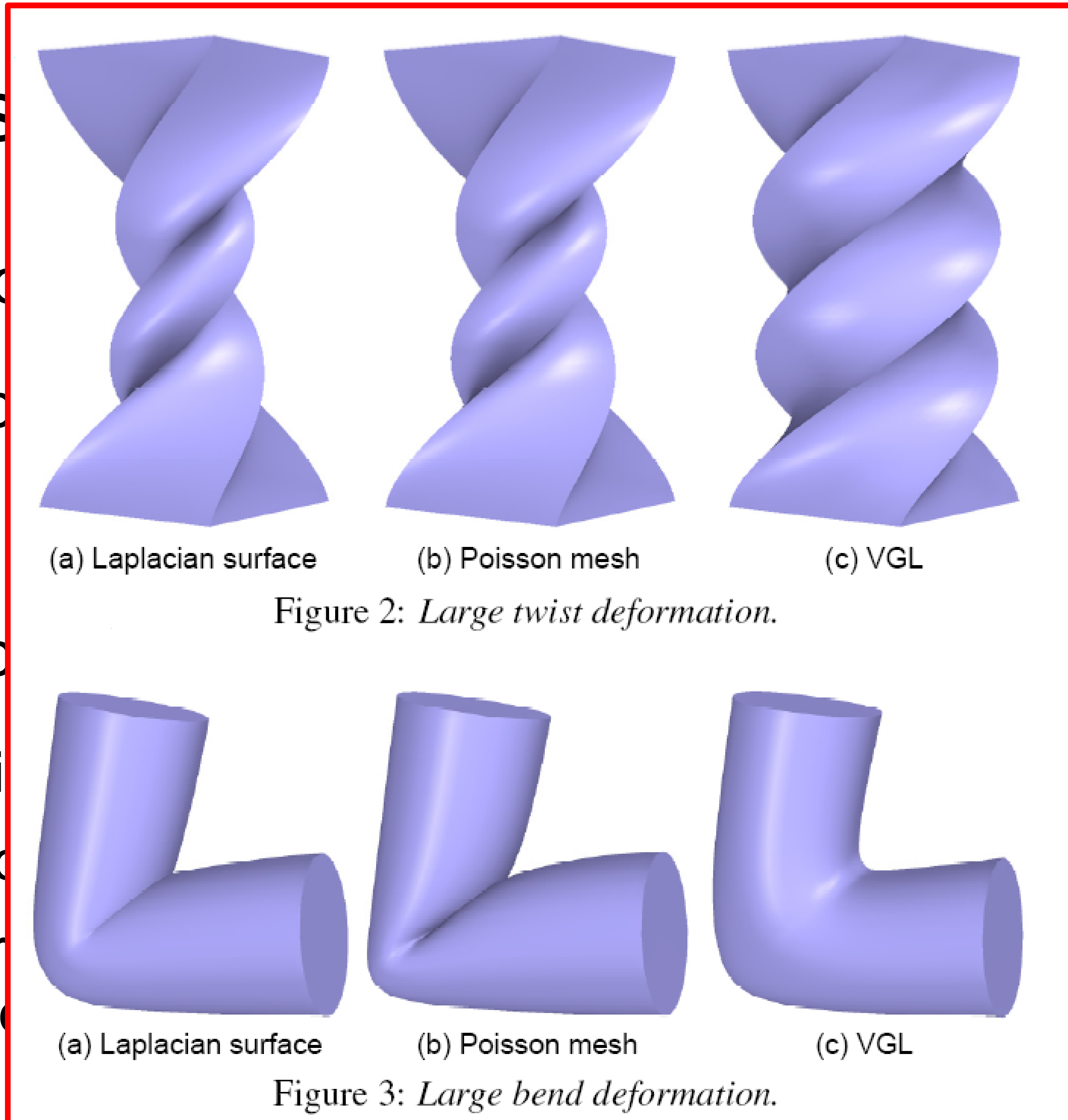

Figure 2: *Large twist deformation.*

Figure 3: *Large bend deformation.*

Constructing the Volumetric Graph

- Algorithm
  - Construct inner shell M_{in} for mesh M by offsetting each vertex a distance in the direction opposite to its Normal
  - Embed M_{in} and M in a body-centered cubic lattice. Remove lattice nodes outside of M_{in}
  - Build edge connections among M, M_{in}, and lattice nodes
  - Simplify the graph using edge collapse and smooth the graph

Constructing the Volumetric Graph

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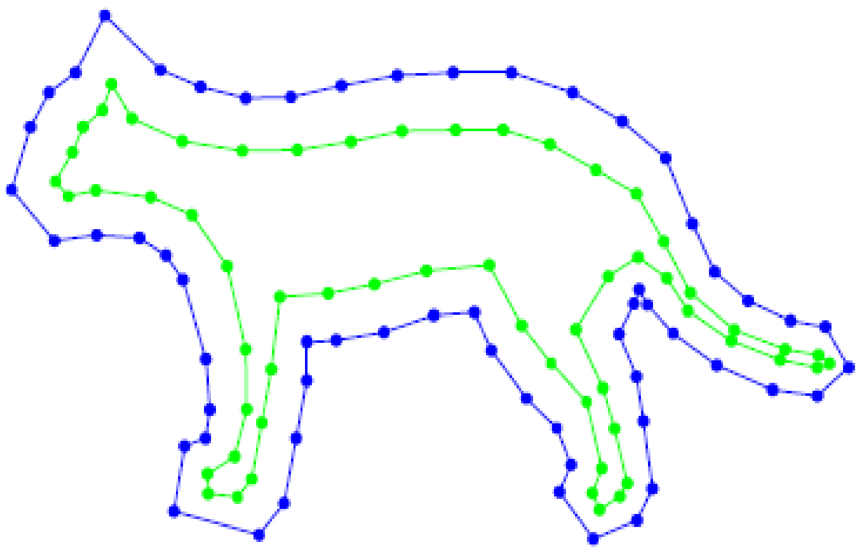
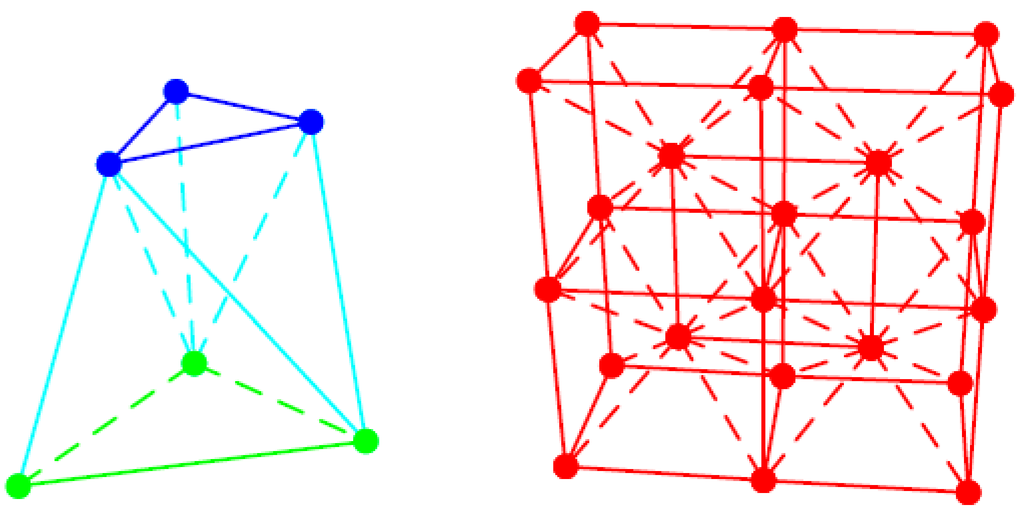


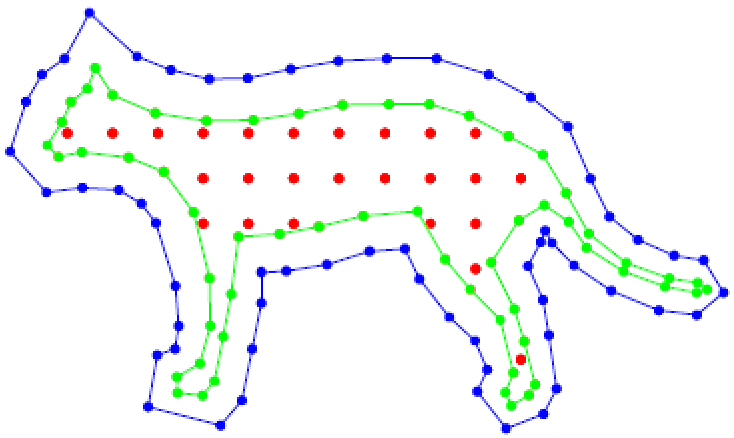
Constructing the Volumetric Graph

- Embed M_{in} and M in a body-centered cubic lattice. Remove lattice nodes outside of M_{in}



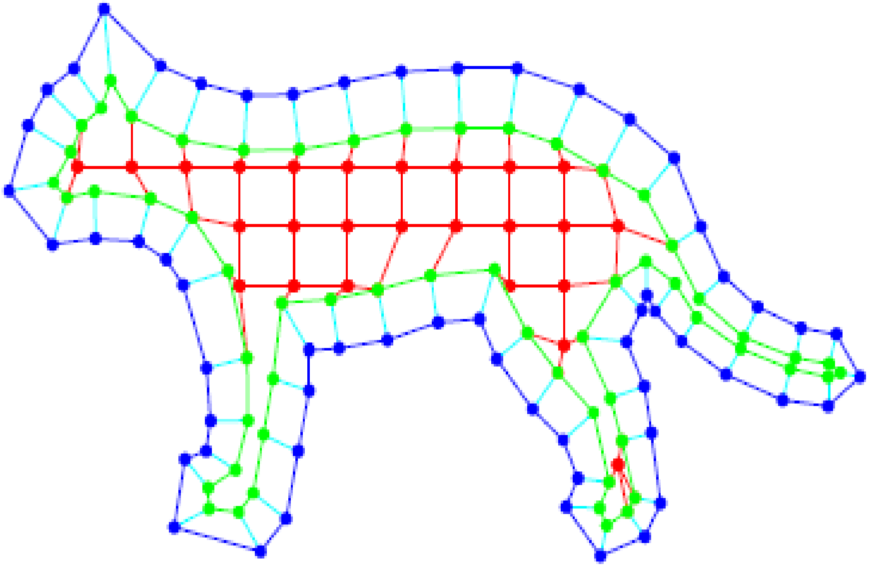
(a) Boundary edges (b) Interior edges

Figure 6: *Types of edge connections in the volumetric graph.*



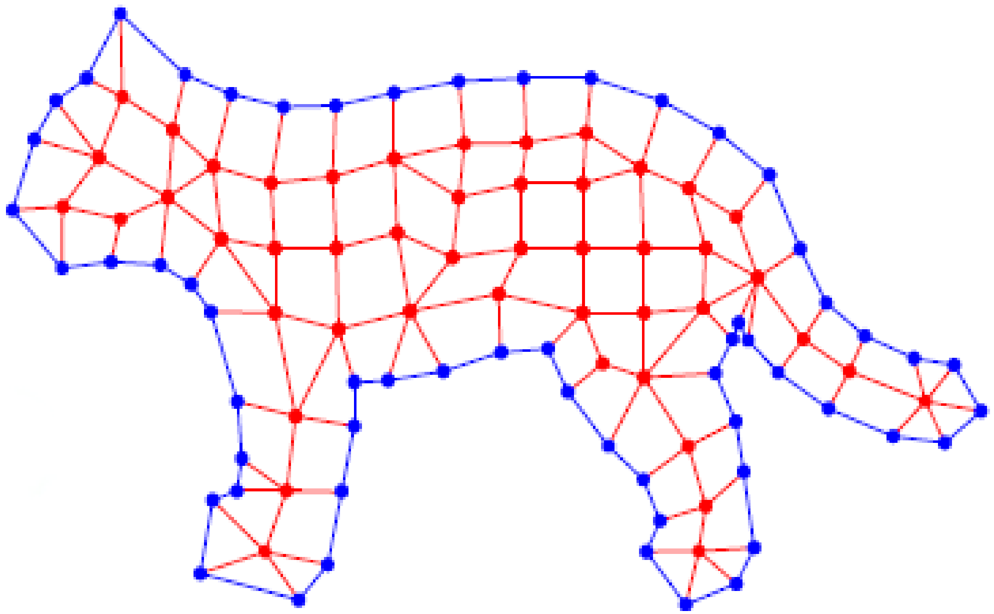
Constructing the Volumetric Graph

- Build edge connections among M, M_{in}, and lattice nodes



Constructing the Volumetric Graph

- Simplify the graph using edge collapse and smooth the graph



Constructing the Volumetric Graph

- M_{in} ensures that inner points are inserted even in thin features that may be missed by lattice sampling.
- Question : how much of a step should one take to construct M_{in}?
- Use iterative method based on simplification envelopes [Cohen et al. 1996]

Constructing the Volumetric Graph

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Constructing the Volumetric Graph

- Use iterative method based on simplification envelopes [Cohen et al. 1996]
  - At each iteration
    - Move each vertex a fraction of the average edge length
    - Test its adjacent triangles for intersection with each other and the rest of the model
    - If no intersections are found, accept step, else reject it
    - Iterations terminate when all vertices have moved desired distance or can no longer move

Constructing the Volumetric Graph

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Constructing the Volumetric Graph

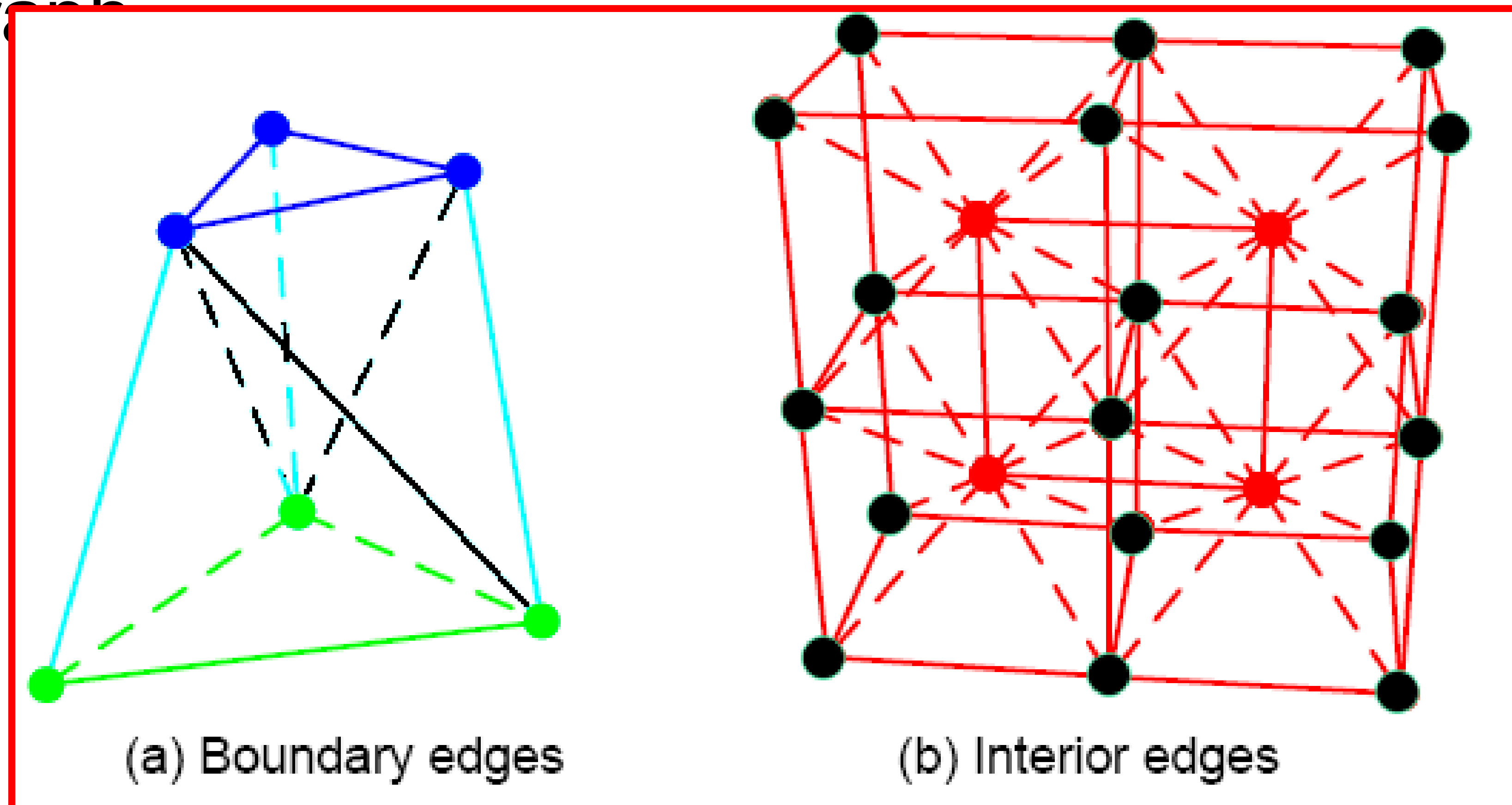
- The BCC lattice
  - Consists of nodes at every point of a Cartesian grid
  - Additionally there are nodes at cell centers
  - Node locations may be viewed as belong to two interlaced grids
  - This lattice provides desirable rigidity properties as seen in crystalline structures in nature
  - Grid interval set to average edge length

Constructing the Volumetric Graph

- Three types of edge connections for an initial graph
  - Each vertex in M is connected to its corresponding vertex in M_{in}. Shorter diagonal for each prism face is included as well.
  - Each inner node of the BCC lattice is connected with its eight nearest neighbors in the other interlaced grid
  - Connections are made between Min and nodes of the BCC lattice.
    - For each edge in the BCC lattice that intersects Min and has at least one node inside M_{in}, we connect the BCC lattice node inside M_{in} to the point in M_{in} closest to this intersection

Constructing the Volumetric Graph

- Three types of edge connections for an initial gr[aph]



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    has at least one node inside Min, we connect the BCC lattice node inside Min to the point in Min closest to this intersection

Constructing the Volumetric Graph

- Simplification and Smoothing
  - Visit graph in increasing order of length
  - If length of an edge is less than a threshold, collapse it to edge's mid point
    - Threshold is half of average edge length of M
  - Apply iterative smoothing
    - Each point is moved to the average of its neighbors
    - Three smoothing operations in their implementation
  - No smoothing or simplification are applied to the vertices of original mesh M

Constructing the Volumetric Graph

- G_{out} can be constructed in a similar way to G_{in}
- M_{out} can be obtained by moving a small step in the normal direction.
- Connections for M_{out} can be made similar M_{in}
- Intersections between M_{in} and M_{out} and with M can occur, especially in meshes containing regions of high curvature.
  - They claim it does not cause any difficulty in our interactive system.

Deforming the Volumetric Graph

- We modify equation (2) to include volumetric constraints

$$\sum_{i=1}^{n} \|\mathscr{L}_M(p_i') - \varepsilon_i'\|^2 + \alpha \sum_{i=1}^{m} \|p_i' - q_i\|^2 + \beta \sum_{i=1}^{N} \|\mathscr{L}_{G'}(p_i') - \delta_i'\|^2 \quad (3)$$

Where the first n points in graph G belong the mesh M

  - G' is a sub-graph of G formed by removing those edges belonging to M
  - δ'_i (1 ≤ i ≤ N) in G' are the graph laplcians coordinates in the deformed frame.
  - For points in the original mesh M, ε' (1 ≤ i ≤ n) are the mesh laplacian coordinates in the deformed coordinate frame

Deforming the Volumetric Graph

- $$\sum_{i=1}^{n} \|\mathcal{L}_M(p'_i) - \varepsilon'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2 + \beta \sum_{i=1}^{N} \|\mathcal{L}_{G'}(p'_i) - \delta'_i\|^2 \quad (3)$$

  - β balances between surface and volumetric detail where $\beta = n\beta'/N$.
  - The n/N factor normalizes the weight so that it is insensitive to the lattice density
  - $\beta' = 1$ works well
  - α is not normalized – We want constraint strength to depend on the number of constrained points relative to the total number of mesh points
    - $0.1 < \alpha < 1$, default is 0.2

Propagation of Local Transforms

- Local Transforms take the Laplacian coordinates in the rest frame to the deformed frame
- Use WIRE deformation method [Singh and Flume]
- Select a sequence of mesh vertices forming a curve
- Deform the curve.

Propagation of Local Transforms

- First determine where neighboring graph points deform to, then infers local transforms at the curve points, finally propagate the transforms over the whole mesh
- Begin by finding mesh neighbors of q_i and obtain their deformed positions using WIRE.
- Let C(u) and C'(u) be the original curve and the deformed curves parametrized by arc length u = [0,1]

Propagation of Local Transforms

- Given some neighboring point p ϵ R_3, let up ϵ [0,1] be the parameter vale minimizing distnace between p and the curve c(u).
- The deformation mapping p to p' such that C maps to C' is given by

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right).$$

- R is a 3x3 rotation matrix taking the tangent vector t(u) on C and maps it to t'(u) on C' by rotating around t(u)xt'(u)

Propagation of Local Transforms

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right).$$

- s(u) is a scale factor
  - Computed at each curve vertex as the ratio of the sum of lengths of its adjacent edges in C' over this length in C
  - It is then defined continuously over u by liner interpolation
- Above equation gives us deformed coordinates for each point in the curve and its 1 Ring neighborhood

Propagation of Local Transforms

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right).$$

- Transformations are propagated from the control curve to all graph points p via a deformation strength field f(p)
- f(p) decays away from the deformation site.
  - Constant
  - Linear
  - Gaussian
  - Based on shortest edge path from p to the curve

Propagation of Local Transforms

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right).$$

- A rotation is defined by
  - Computing a normal and tangent vector as the perpendicular projection of one edge vector with this normal
  - Normal is computed as a linear combination weighted by face area of face normals around mesh point i
  - Rotation is represented as a quaternion
    - Angle should be less than 180

Propagation of Local Transforms

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right).$$

- The simplest propagation scheme is to assign to p a rotation and scale from the point q_p on the control curve closest to p
- Smoother results are obtained by computing a weighted average over all the vertices on the control curve instead of the closest

$$\exp\left(-\frac{(\|p-q_i\|_g - \|p-q_p\|_g)^2}{2\sigma^2}\right)$$

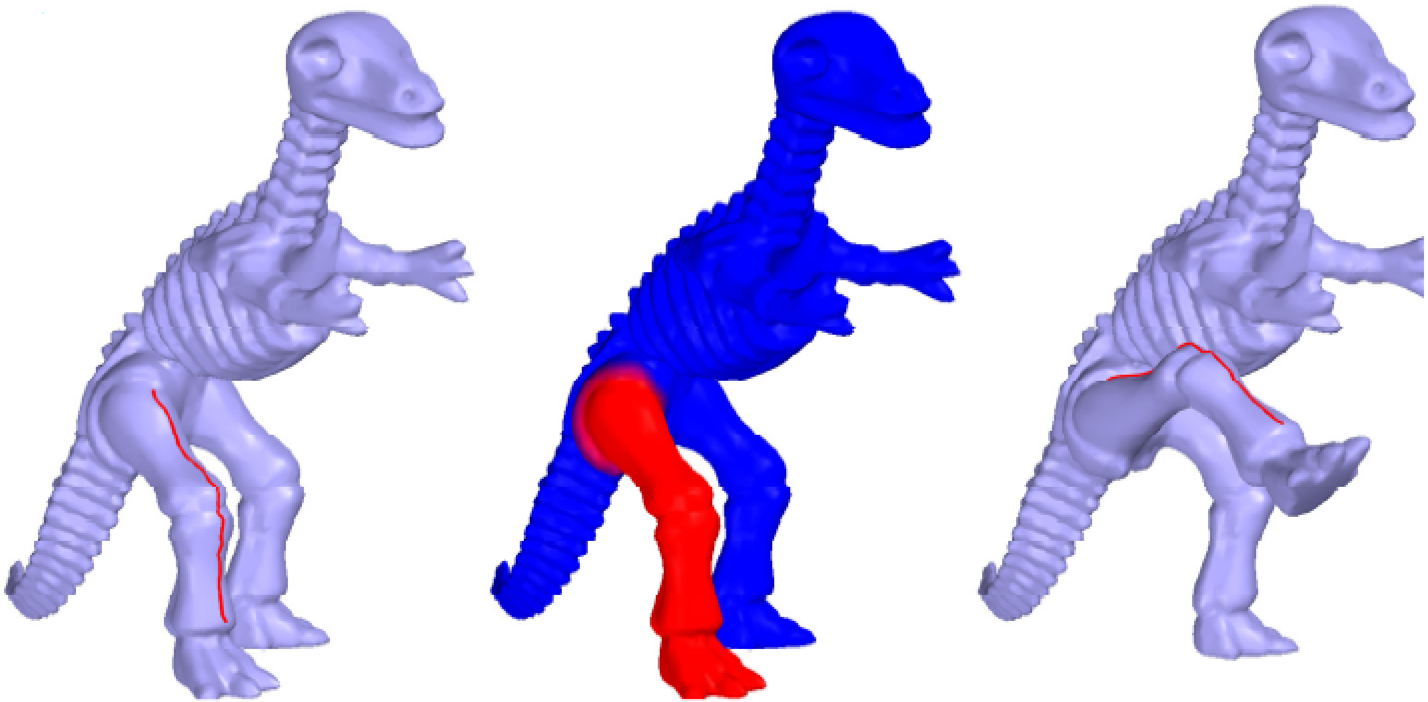
Propagation of Local Transforms

$$p' = C'(u_p) + R(u_p)\left(s(u_p)(p - C(u_p))\right).$$

- Weighting over multiple curves is similar, we accumulate values over multiple curves
- Final transformation matrix is given by

$$T_p = f(p)\tilde{T}_p + (1 - f(p))I$$

Propagation of Local Transforms



Weighting Scheme

- They drop uniform weighting in favor of another scheme that provides better results
- For mesh Laplacian L_m, use cotangent weights

$$w_{ij} \propto (\cot \alpha_{ij} + \cot \beta_{ij}),$$

where $\alpha_{ij} = \angle(p_i, p_{j-1}, p_j)$ and $\beta_{ij} = \angle(p_i, p_{j+1}, p_j)$.

- For graph Laplacian, compute weights by solving a quadratic programming problem

Weighting Scheme

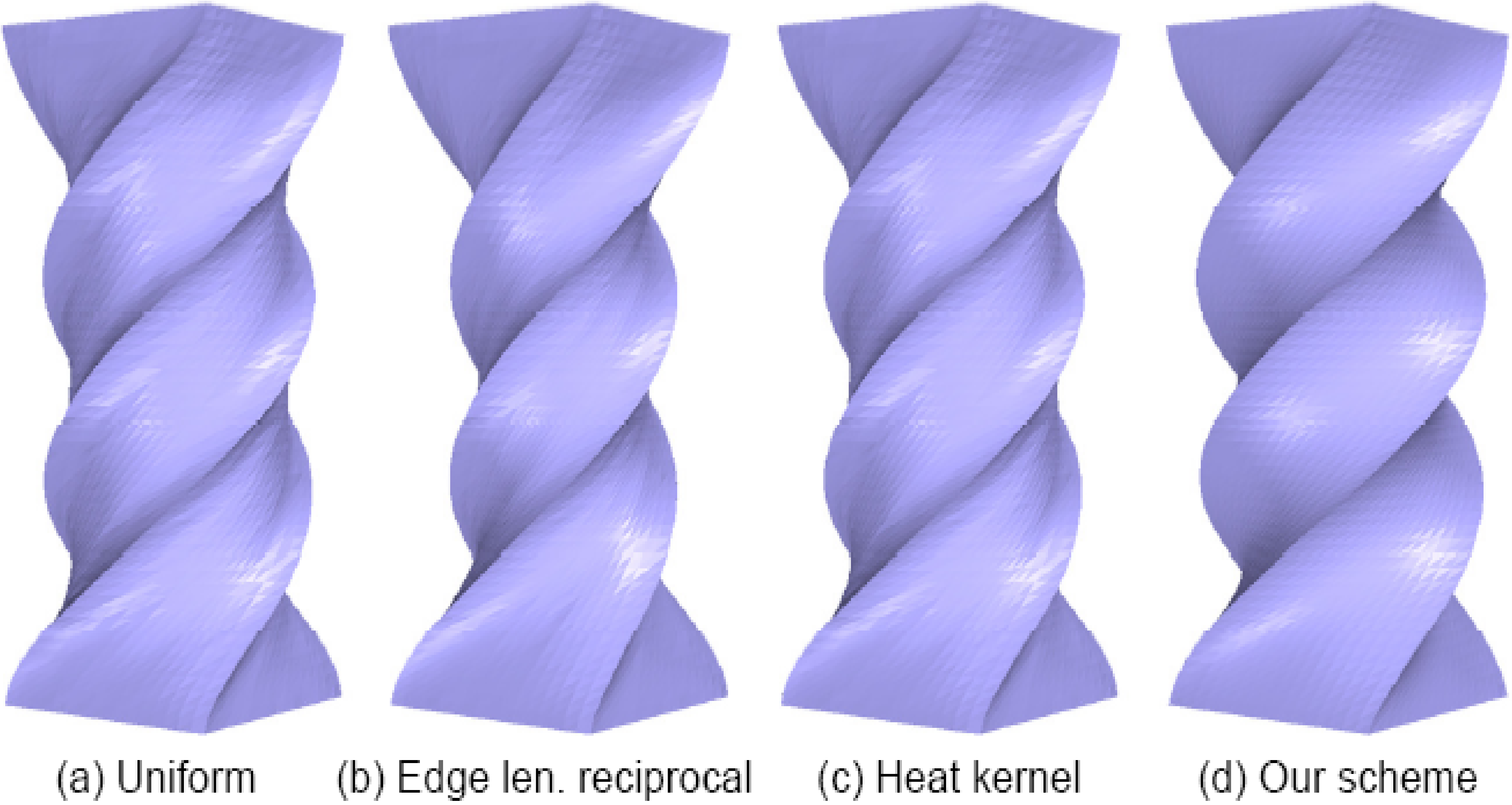
- For each graph vertex i, to obtain weights w_{ij} solve

$$\min_{w_j}\left(\|p_i - \sum_{j\in\mathcal{N}(i)} w_j p_j\|^2 + \lambda\left(\sum_{j\in\mathcal{N}(i)} w_j \|p_i - p_j\|\right)^2\right)$$

$$\text{subject to} \quad \sum_{j\in\mathcal{N}(i)} w_j = 1 \text{ and } w_j > \xi .$$

  - The first term generates Laplacian coordinates of smallest magnitude
  - Second term is based on scale dependent umbrella operator which prefers weight in proportion to inverse of edge length
  - Lamba balances the two objects (set to 0.01)
  - Zeta prevents small weights (set to 0.01)

Weighting Scheme



Quadric Energy Minimization

- To minimize energy in equation (3) we solve the following equations

$$\mathscr{L}_M(p_i') + \beta\, \mathscr{L}_{G'}(p_i') = \varepsilon_i' + \beta\, \delta_i', \quad i \in 1, \ldots, n,$$

$$\beta\, \mathscr{L}_{G'}(p_i') = \beta\, \delta_i', \quad i \in n+1, \ldots, N,$$

$$\alpha\, p_i' = \alpha\, q_i', \quad i \in 1, \ldots, m$$

  - Above equations represent a sparse linear system Ax = b
  - Matrix A is only dependent on the original graph and A^- can be precomputed using LU decomposition
  - B depends on current Laplacian coordinates and changes during interactive deformation

Multi resolution Methods

- Solving a the linear system of a large complex model is expensive
- Generate a simplified mesh [Guskov et al. 1999]
- Deform this mesh and then add back the details to obtain high resolution deformed mesh

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Deformation from 2D curves

- Method
  - User defines control curve by selecting sequence points on the mesh which are connected by the shortest edge path (dijkstra)
  - This 3D curve is projected onto one or more planes
  - Editing is done in these planes
  - The deformed curve is projected back into 3D, which then forms the basis of the deformation process

Deformation from 2D curves

- Curve Projection
  - Given a curve, the system automatically selects projection planes base on its average normal and principal vectors.
  - Principal vectors are computed as the two eigen vectors corresponding to the largest eigen values from a principal component analysis
  - In most cases, cross product of the average normal and the first principal vector provide a satisfactory plane
  - If length of average normal vector is small, then use only two principal vectors instead

Deformation from 2D curves

- Curve Editing
  - Projected 2D curves inherit geometric detail from original mesh that complicates editing
  - They use an editing method for discrete curves base on Laplacian coordinates
  - Laplacian coordinate of a curve vertex is the difference between its position and the average position of its neighbors or a single neighbor in cases of terminal vertices
  - Denote the 2D curve to be edited as C
  - A cubic B-Spline curve C_b is first computed as a least squares fit to C. This represents the low frequencies of C

Deformation from 2D curves

- Curve Editing
  - A discrete version of C_b , C_d is computed by mapping each vertex of C onto C_b using proportional arc length mapping
  - We can not edit the discrete version conveniently
  - After editing we obtain C'_b and C'_d . These curves lack the original detail of the C_b
  - To restore detail, at each vertex of C we find a the unique rotation and scale that maps its location from C_d to C'_d
  - Applying these transformations to the Laplacian coordinates and solving equation (2) without the constraint term

$$\min_{p'_i} \left(\sum_{i=1}^{N} \|\mathscr{L}_G(p'_i) - \delta'_i\|^2 + \alpha \sum_{i=1}^{m} \|p'_i - q_i\|^2 \right). \qquad (2)$$

Deformation from 2D curves

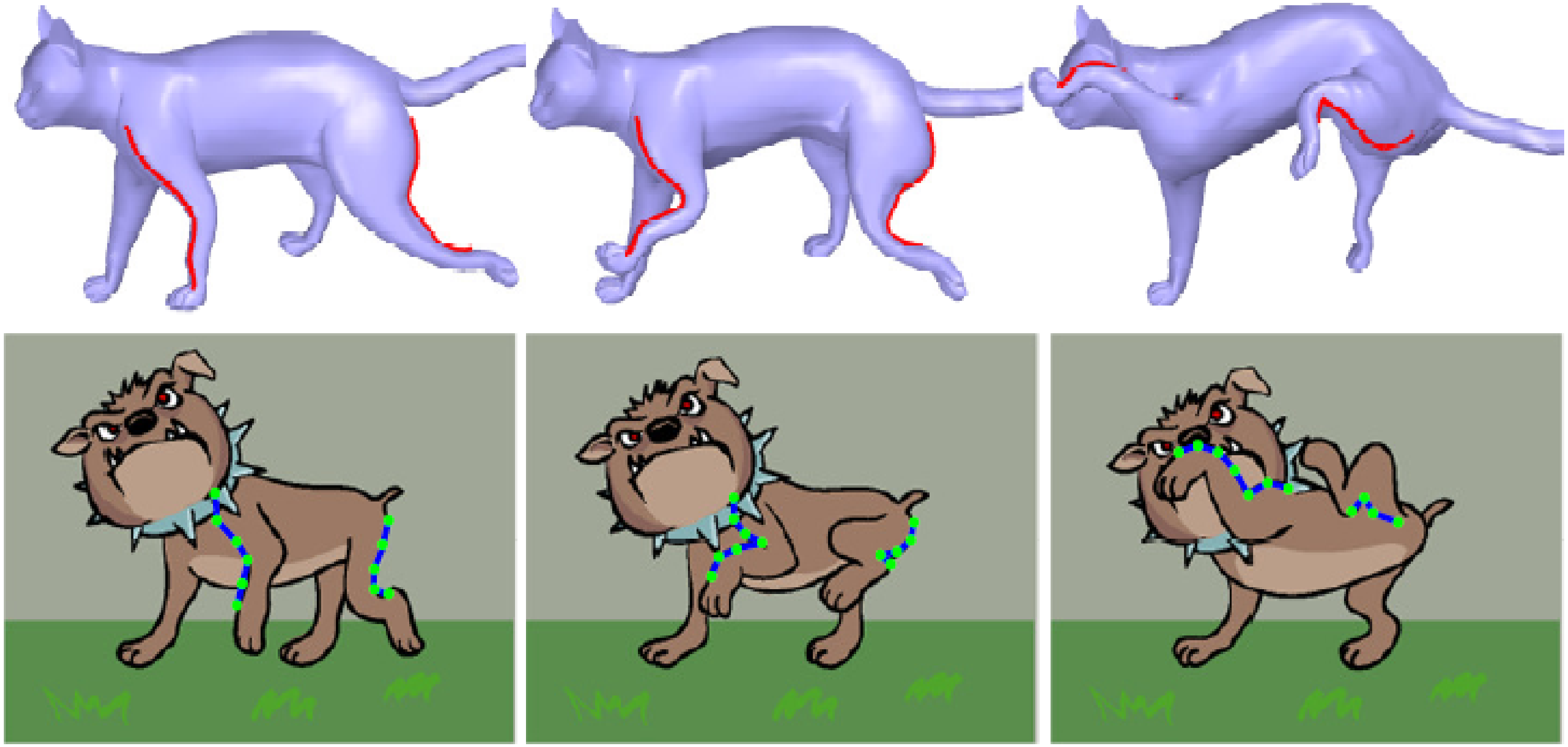
- Deformation Re-targeting from 2D Cartoons
  - An application of 2D sketch based deformation
  - Users specify one or more 3D control curves on the mesh along with their project planes and for each curves a series of 2D curves in the cartoon image sequence that drive its deformation
  - It is not necessary to generate a deformation from scratch at every time frame. Users can select a curves in a few key frames of the cartoon
  - Automatic interpolation technique based on differential coordinates is used to interpolate between key frame

Deformation from 2D curves

- Deformation Re-targeting from 2D Cartoons
  - Say we have two meshes M and M' at two different key frames
  - Compute the Laplacian coordinates for each vertex in the two meshes
  - A rotation and scale in the local neighborhood of each vertex p is computed taking the Laplacian coordinates from its location in M to M'
  - Denote the transform as T_p. Interpolate T_p over time to transition from M to M'
  - 2D cartoon curves are deformed in a single plane, this allows for extra degrees of freedom if required by the user

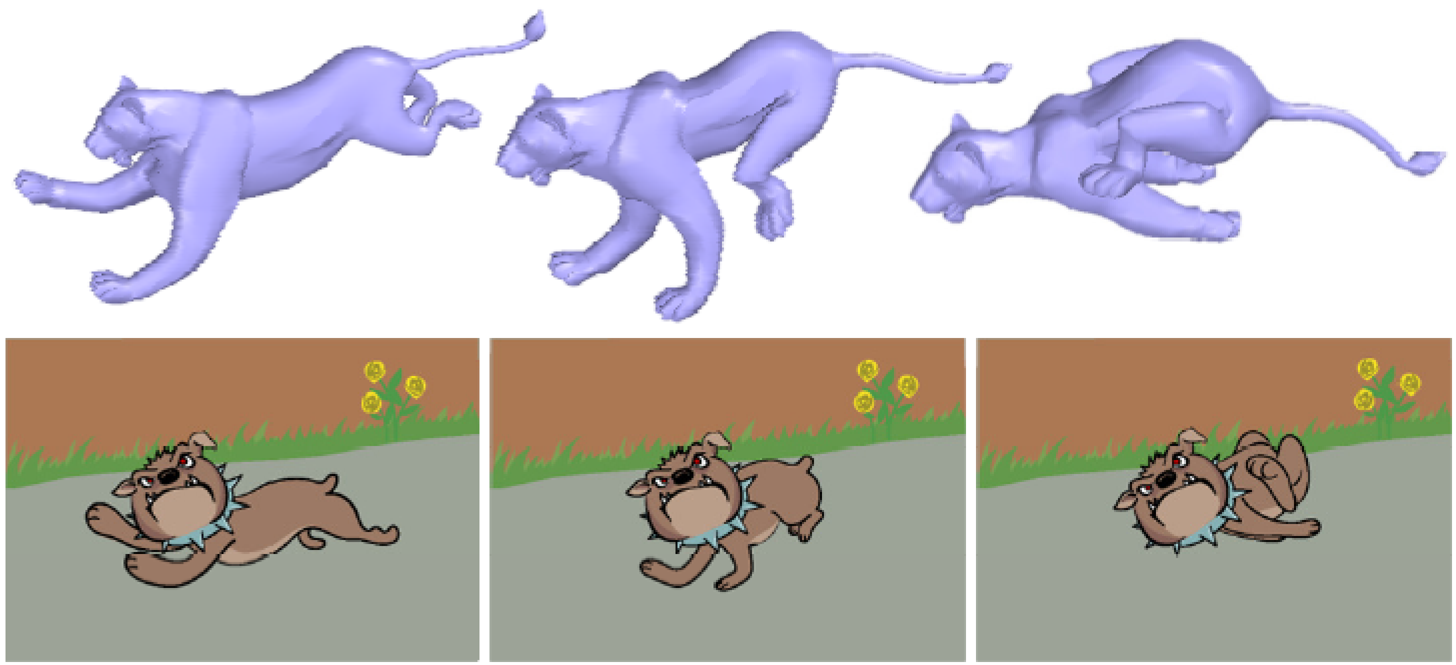
Deformation from 2D curves

- Deformation Re-targeting from 2D Cartoons



Deformation from 2D curves

- Deformation Re-targeting from 2D Cartoons



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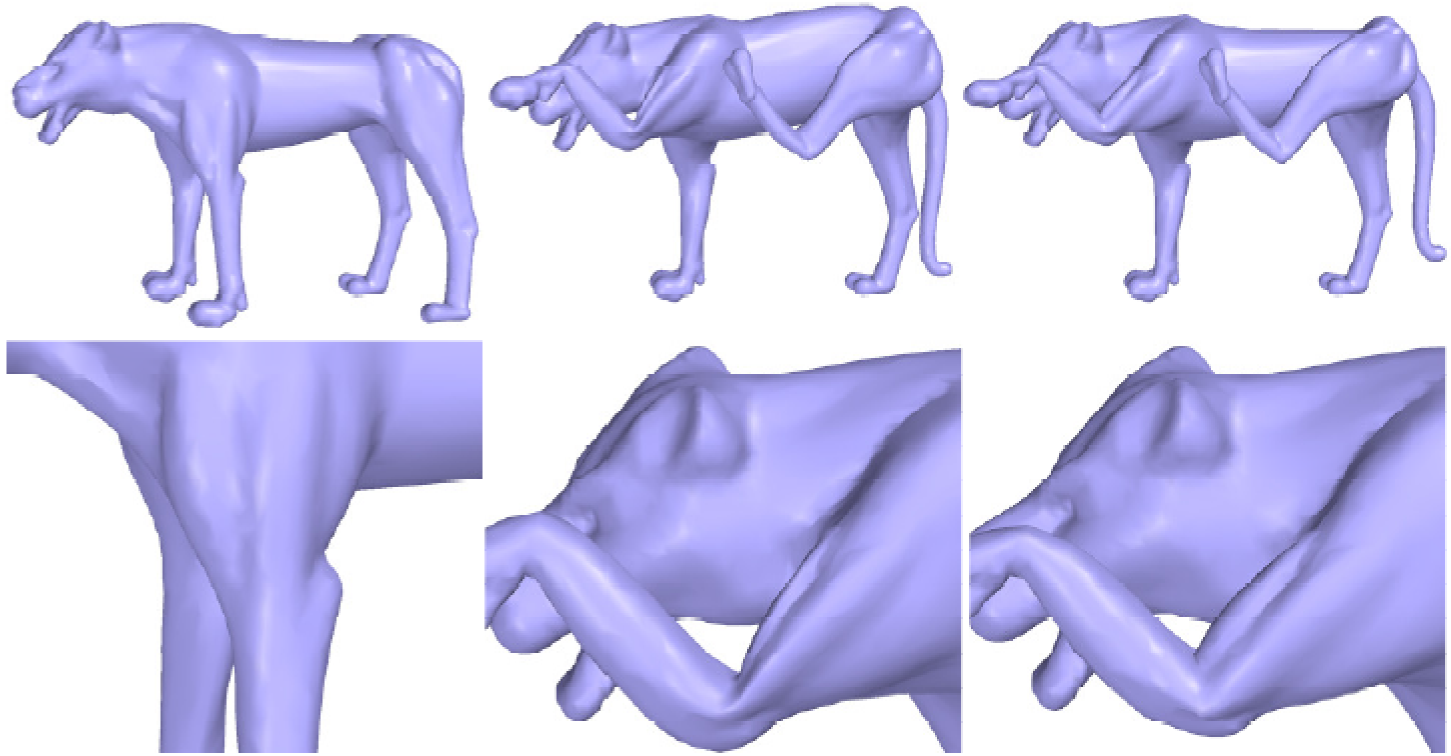
Results

- Some stats

	arma	dino	cat	lioness	dog
# mesh vertices	15,002	10,002	7,207	5,000	10,002
# graph points	28,142	15,895	14,170	8,409	17,190
graph generation	2.679s	1.456s	1.175s	1.367s	1.348s
LU decomposition	0.524s	0.286s	0.348s	0.197s	0.118s
back substitution	0.064s	0.028s	0.030s	0.019s	0.011s
# control curves	6	5	4	5	
# key frames	10	9	8	8	
session time (min)	∼120	∼90	∼30	∼90	

Table 1: *Statistics and timing.*

More results



(a) Original mesh (b) Poisson mesh editing (c) VGL

More results

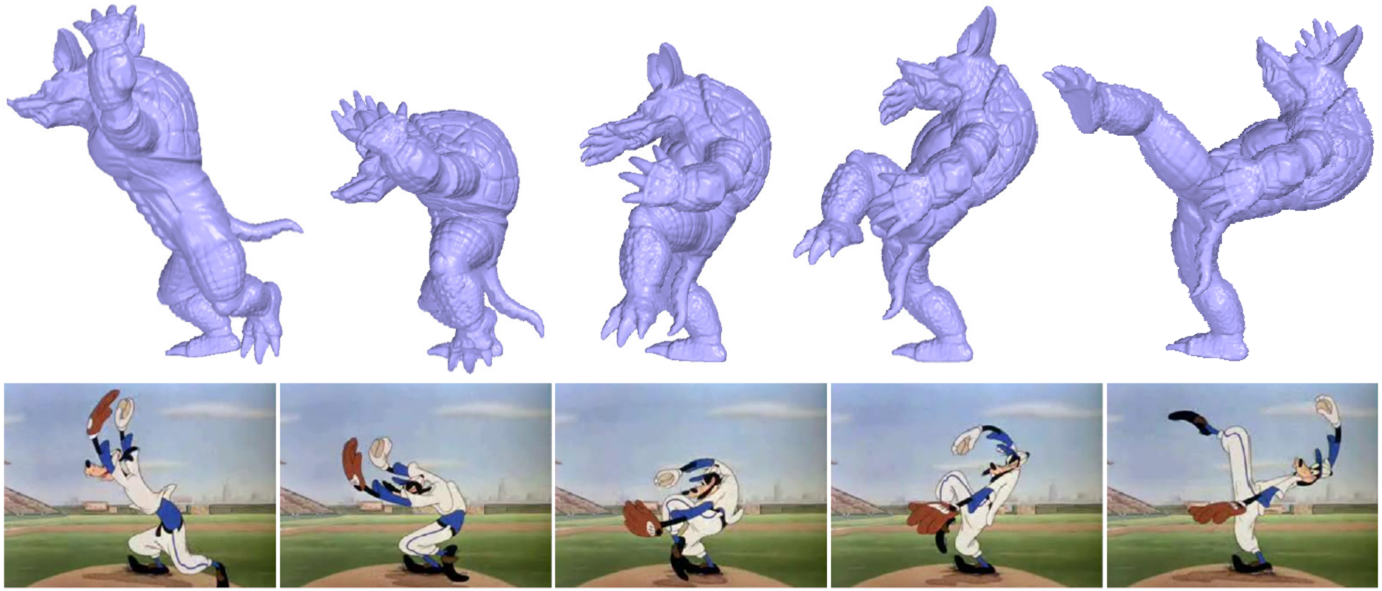


Figure 12: *Deformation transfer from Goofy to armadillo.* ©*Disney*

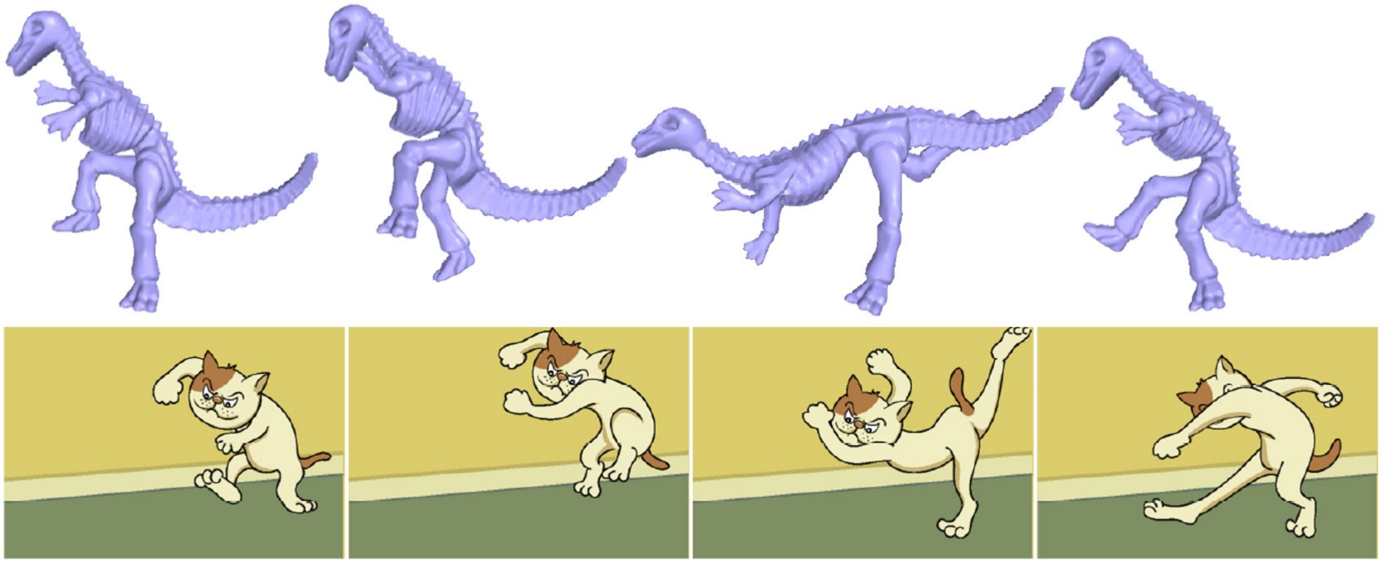


Figure 13: *Deformation transfer from a kicking cat to dinosaur.*

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Conclusions

- They proposed a system which would address volumetric changes and local self intersection based on the volumetric graph Laplacian
- The solution avoids the intricacies of solidly meshing complex objects
- Presented a system for retargetting 2D animations to 3D
- Note, that their system does not address global self intersections – those must addressed by the user