A Signal Processing Approach to Fair Surface Design

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Overview
- Introduction
- Fairing
- Subdivision
- Constraints
- Conclusions

Introduction
- Motivation
  - Fairing surfaces
  - Medical data
- Goals
  - Quick, linear-time algorithm
  - Non-shrinking
  - Incorporate constraints

Closed Curve Fairing
- Extension of Fourier analysis to surfaces
- Discrete Laplacian for surface
  \[ \Delta w_i = \frac{1}{2}(w_{i-1} - w_i) + \frac{1}{2}(w_{i+1} - w_i) \]
  \[ \Delta x = -Kx \]
  \[ K = \frac{1}{2} \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \]
  \[ w = \sum_{i=1}^{n} \xi_i w_i \]
Closed Curve Fairing

- Unfortunately the decomposition of this summation is expensive even with FFT
- Instead, lets do this projection approximately
  \[ w'_i = w_i + \lambda \Delta w_i \]
  \[ x' = (I - \lambda K) x \]
- As Gaussian filtering, this produces shrinkage.
- They define a non-shrinking function to replace \( I - \lambda K \)

Surface Signal Fairing

- Discrete surface signal
  \[ \Delta w_i = \sum_{j \in \Gamma} w_{ij}(x_j - x_i) \]
- Choosing weights
  \[ w_{ij} = \frac{\phi(v_i, v_j)}{\sum_{k \in \Gamma} \phi(v_i, v_k)} \]
- While Fourier descriptors are no longer feasible for large spaces of vertices, low-pass filtering still is.

Surface Signal Fairing

- Polynomial transfer function
  \[ x' = f(K)x = \sum_{i=1}^{n} \xi_i f(k_i) w_i \]

Surface Signal Fairing

- Low-pass filter
  \[ f(k) = (1 - \lambda k)(1 - \mu k) \]
- Pass-band frequency
  \[ k_{pn} = \frac{1}{\lambda} + \frac{1}{\mu} > 0 \]
**Surface Signal Fairing**

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**Surface Signal Fairing**

vertex \( v_i \)

neighbors \( v_j : j \in i^* \)

new position \( v'_i = v_i + \{\lambda, \mu\} \sum_{j \in i^*} w_{ij} (v_j - v_i) \)
Surface Signal Fairing

Subdivision
- Subdivision schemes suffer from shrinkage.
- Using this smoothing method, shrinkage is reduced.

Interpolatory Constraints
- Easy constraint is non-symmetric neighborhoods

Smooth Interpolation
- Instead of simply editing the neighborhood, they add in a new deformation that returns a point to its original location
  - \( \tilde{\mathbf{a}}^N = \mathbf{a}^N + (\mathbf{a}_1 - \mathbf{a}_1^N) \mathbf{d}_1 \)
  - More generally:
  - \( \tilde{\mathbf{a}}^N = \mathbf{a}^N + \mathbf{F}_{mm}^{-1} \begin{pmatrix} \mathbf{a}_1 - \mathbf{a}_1^N \\ \vdots \\ \mathbf{a}_m - \mathbf{a}_m^N \end{pmatrix} \)
  - Computation can be sped up by precomputing
  - \( \mathbf{F}_{mm} \begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_m \end{pmatrix} = \begin{pmatrix} \mathbf{a}_1 - \mathbf{a}_1^N \\ \vdots \\ \mathbf{a}_m - \mathbf{a}_m^N \end{pmatrix} \)
**Smooth Deformations**

**Hierarchical Constraints**

**Tangent Plane Constraints**

- Shading
  \[ \Delta v_i = \sum_{j \in \mathcal{E}} w_{ij}(v_j - v_i) \]
  \[ \frac{1}{|\mathcal{E}|} \int_{v \in \mathcal{E}} (v - v_i) dl(v) \]
  \[ \lim_{\Delta \to 0} \frac{1}{|\mathcal{E}|} \int_{v \in \mathcal{E}} (v - v_i) dl(v) = N(v_i) \]
  \[ B_i \Delta v_i^N = T_i \Delta v_i^N = 0 \]

**General Linear Constraints**

- Generalization of the smooth interpolation problem seen before.
  \[ Cx_\mathcal{E} = c \]
- To see it is equivalent to smooth interpolation we look at:
  \[ \begin{cases} u_{(1)}(1) = u(1) - C_{(1)}^{-1} C_{(2)} w(2) \\ u_{(2)} = w(2) \\ y(1) = C_{(1)}^{-1} c \end{cases} \]
Conclusions

- Linear time and space fairing system
- Constraints allowed
  - Differing network structure
  - Differing vertex handling
  - Subdivision

Questions?