Shape Representation and Classification Using the Poisson Equation

Gorelick L. Galun M. Sharon E.
Ronen B. and Brandt A.

Presented by
Raphael Sznitman

Overview

- Introduction
- Methodology
  - Poisson Equation and solution
  - Feature Extraction
- Applications and Experimentation
  - Hand Written Numerals
  - Natural Silhouettes

Introduction

- Silhouette contours contain detailed information about the shape of objects.
- Interested in having a way to extract features from the given silhouettes.
- Features looked at here:
  - Corners
  - Skeleton
  - Local orientation
  - Hierarchical Shape

Methodology: Poisson Equation

- At every point (x,y) in the silhouette, S , calculate the expected time necessary to reach the boundary when doing a random walk.
- Can be computed by:

$$\Delta U(x, y) = -1$$

where $\Delta U = U_{xx} + U_{yy}$ subject to the boundary conditions $U(x, y) = 0$ at the boundary contour ∂S

Methodology: Poisson Equation

- The solution to this equation gives us different level sets of U.
- Use the Multigrid solution to solve the Poisson equation.

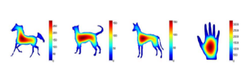


Figure 2: Solutions to the Poisson equation for the silhouettes in Fig. 1.

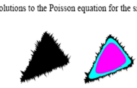


Figure 3: Noisy boundary (left) and two level sets of low values (right).

Methodology: Feature Extraction

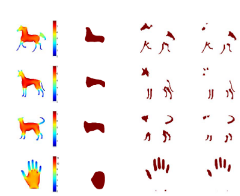
- Hierarchical Shape

Goal is to find central piece of a silhouette.

This is done by thresholding large values of U and adding:

$$\Phi = U + U_x^2 + U_y^2$$

in order to account for central border pieces.



Methodology: Feature Extraction

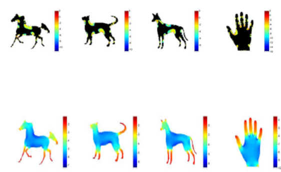
- Corners

Evaluate curvature at a point in S by:

$$\Psi = -\nabla \cdot \frac{\nabla U}{\|\nabla U\|}$$

High positive values of will give convex regions along with ridge. Good for detecting corners at different scale

Although the best results when using low values of Φ



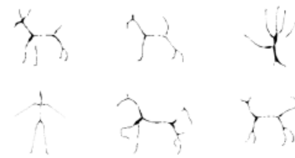
Methodology: Feature Extraction

- Skeleton

Use a scale-invariant versions of Ψ:

$$\hat{\Psi} = -\frac{U \cdot \Psi}{\|\nabla U\|}$$

Pick small values of U to get silhouette skeleton.



Methodology: Feature Extraction

- Local Orientation

Simple Case

Split silhouette into sub parts depending on the local pixel orientation.

The orientation of a given pixel which has small $U_{\theta\theta}$ for a given θ.

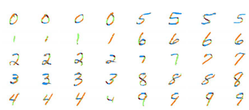


Figure 8: A schematic representation of the second derivatives for hand-written numerals: each pixel is assigned a color according to the direction in which the derivative at this pixel is minimal (indicating that the pixel lies in a part that is elongated in this direction), i.e. green for the vertical direction, blue for horizontal, brown and orange for the two diagonals.

Methodology: Feature Extraction

- Local Orientation

Complex case

Construct measure of second order moments of a shape near a given point.

This is done by getting the Hessian matrix of U at that point.

Eigenvectors and eigenvalues reveal orientation, aspect ratio and if shape is elliptical/hyperbolic

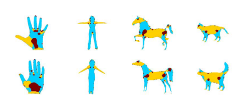


Figure 10: A schematic representation of orientations computed via the Hessian matrix for several shapes. Turquoise color represents vertical regions ($|\alpha| \geq \frac{\pi}{4}$), yellow for horizontal ($|\alpha| < \frac{\pi}{4}$), and brown for isotropic sections $\sqrt{|\lambda_{\min}/\lambda_{\max}|} \geq \frac{2}{3}$). It can be seen that pairs of similar shapes give rise to similar orientation measures, but notice the thumb (left figures), which changes its orientation from near horizontal (top) to near vertical (bottom).

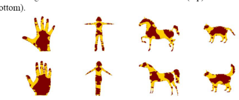


Figure 11: Elliptic (brown) and hyperbolic (yellow) sections computed with the Hessian matrix for several shapes (sign of $\lambda_{\min} \cdot \lambda_{\max}$).

Application

- Performed classification experiments using these extracted features. Features are weighted moments:

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} w(x,y) x^p y^y \, dx dy$$

where w(x,y) is some Poisson base measure.

- Use Decision trees as classifier.
- Compared results to literature.

Application

- Hand-Written Numerals
  Used two types of measures for weights:
  - Local Orientation
  - Junctions
- Natural Silhouettes
  Also used two types of measures for weights:
  - Vertical and Horizontal regions where orientations are close to 0 or $\pi/2$ (using Hessian matrix)
  - Concave regions using Φ