3D Scattered Data Approximation with Adaptive Compactly Supported Radial Basis Functions

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Problem Statement

- Given a set of points, $\mathcal{P}=\{\mathbf{p}_1, \ldots, \mathbf{p}_N\}$, and corresponding unit normals, $\mathcal{N}=\{\mathbf{n}_1, \ldots, \mathbf{n}_N\}$, find a function f such that the zero set $f(\mathbf{x}) = 0$ approximates $\mathcal{P}$.
- NOTE: This definition of f, given in the text, does not regard $\mathcal{N}$ and does not address the trivial solution.

Solution Outline

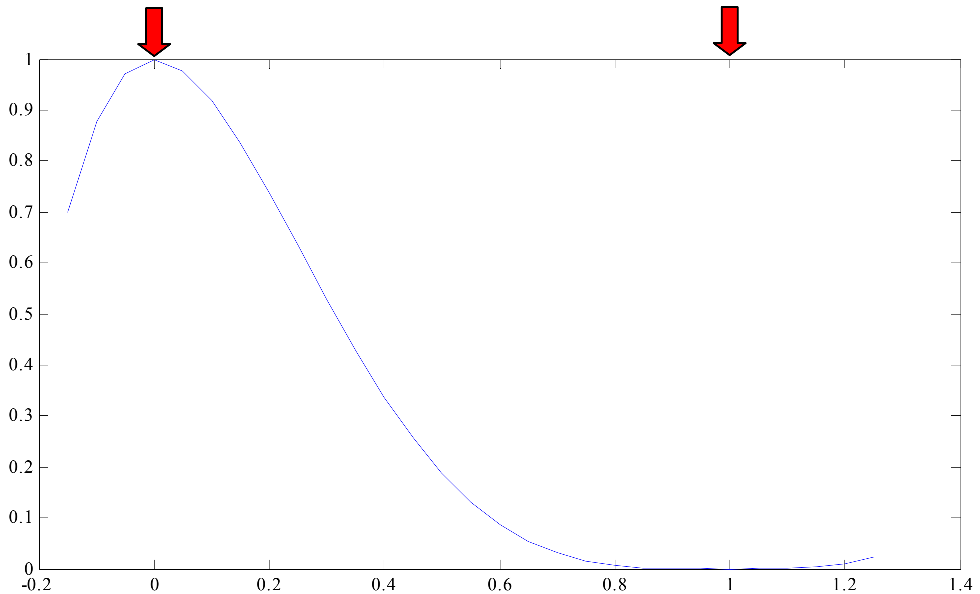
- Use a set of *approximation centers* $\mathcal{C} = \{\mathbf{c}_1, \ldots, \mathbf{c}_M\}$
- Approximate $\mathcal{P}$ by f as follows:

$$f(\mathbf{x}) = \sum_{c_i \in \mathcal{C}} \left[g_i(\mathbf{x}) + \lambda_i \right] \phi_{\sigma_i}\left(\|\mathbf{x} - \mathbf{c}_i\| \right)$$

- σ's are influence radii for the centers
- g's are unknown functions, λ's are unknown coefficients
- ϕ's are *radial basis functions*.

$$\phi(r) = (1-r)_+^4 (4r+1)$$

Radial Basis Function



$$\phi(r) = (1-r)_+^4 (4r+1)$$

Solution Outline

- Construct g_i as a local quadratic approximation of $\mathcal{P}$ in $\{\|\mathbf{x}\text{-}\mathbf{c}_i\| < \sigma_i\}$
- Find λ's through solving a linear equation: $f(\mathbf{c}_j) = 0$
- Normalize the basis functions and decompose f

$$\underbrace{\sum_{\mathbf{c}_i \in \mathcal{C}} g_i(\mathbf{x})\Phi_{\sigma_i}(\|\mathbf{x}-\mathbf{c}_i\|)}_{\text{adaptive PU}} + \underbrace{\sum_{\mathbf{c}_i \in \mathcal{C}} \lambda_i \Phi_{\sigma_i}(\|\mathbf{x}-\mathbf{c}_i\|)}_{\text{normalized RBF}} = 0$$

Main Properties

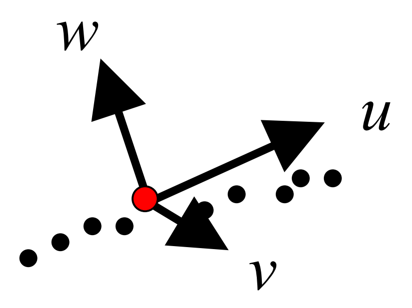
- Reduction of problem size by using a small number of approximation centers.
- RBF's on a limited domain ➔focus on local features
- Different influence radii for different approximation centers

Explaining Equation 7.5

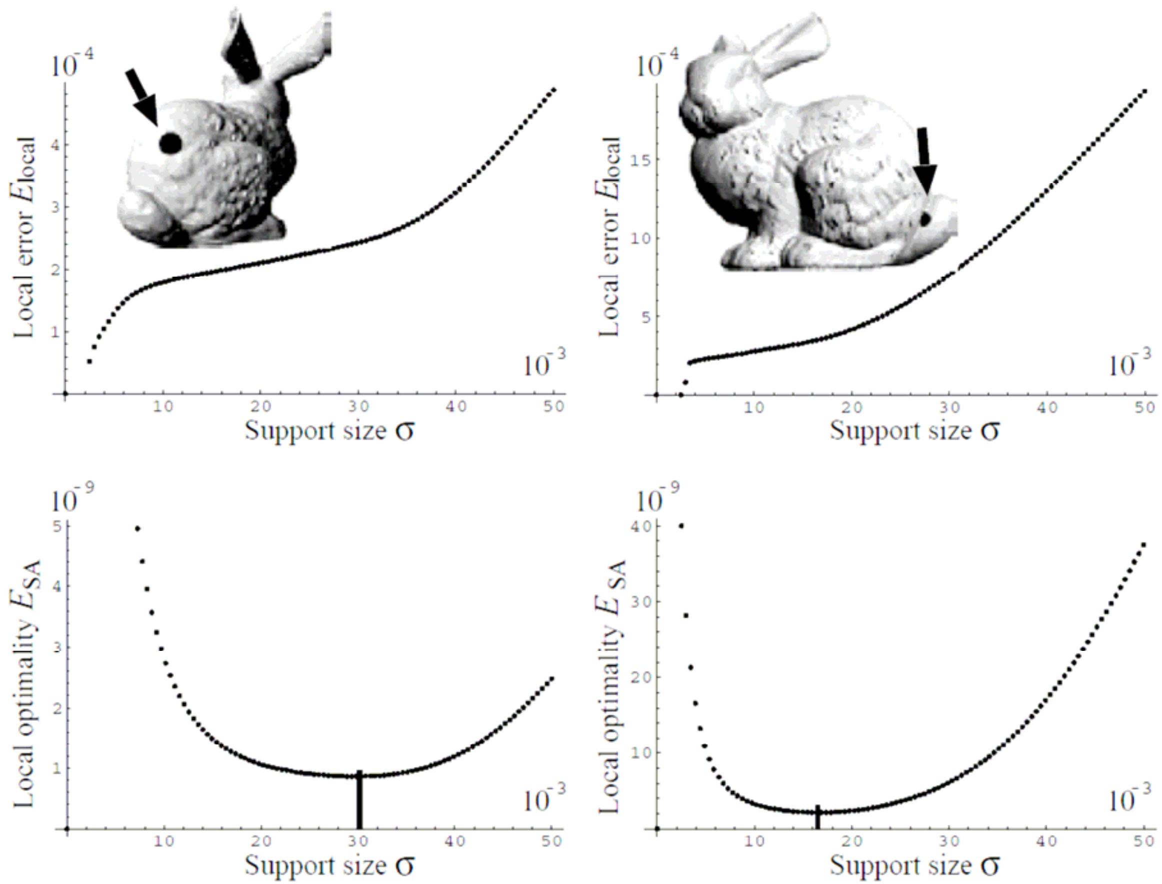
- h is a local approximation of the surface as a bivariate quadratic function on the tangent plane at $\mathbf{p}_i$.
  - $g_i(\mathbf{x}) = w - h(u, v)$
- w is measured for each neighbor of $\mathbf{p}_i$ as the distance from the tangent plane.

$$w = h(u, v) \equiv Au^2 + 2Buv + Cv^2 + Du + Ev + F$$

- The coefficients are computed by solving linear equations.



Finding Local Influence Radius



$$E_{\text{SA}}(\sigma) = E_{\text{local}}(\sigma)^2 + \frac{C}{\sigma^2},$$