Spectral Surface Reconstruction
from Noisy Point Clouds

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December 5th, 2005

Some Definitions

- \( S \) = sample points
- \( S^+ \) = \( S \) augmented with bounding box vertices
- \( T \) = Delauney Tetrahedralization of \( S^+ \)
- \( Q \) = Voronoi Diagram of \( S^+ \)

For each tetrahedron \( t \) in \( T \), there is a dual voronoi vertex \( v \) in \( Q \).

Goal:

Label each tetrahedron—or equivalently, each voronoi vertex—inside or outside

Identify the special Voronoi vertices called poles.

Recall poles are voronoi vertices that are likely to lie near the medial axis of the surface being recovered. A sample point \( S \) can have 2 poles.

Form graph \( G = (V, E) \)

\( V \) = voronoi vertices.
\( E \) = edges.

For each point \( s \) with poles \( u, v, (u, v) \) is an edge in \( E \).

For pairs of samples \( s \) and \( s' \) such that \((s, s')\) is edge in \( T \), where \( s \) has poles \( u, v \) and \( s' \) has poles \( u', v' \), then include edges \((u, u'), (u, v'), (v, u')\) and \((v, v')\).
Spectral Surface Reconstruction

Form graph $G' = (V, E)$

Collapse certain poles into single node which are known to be "outside".

- Any tetrahedron with a vertex of the bounding box
- Direction from laser range finder
- Visual inspection

Spectral Based Partitioning

Form $G'$ and construct a Laplacian matrix $L$:

$$\sigma = \sum_{j=1}^{n} \left| L_{ij} \right|$$

Laplacian Spectrum Partitioning

Define $x$ as a partitioning vector of all the nodes

$$x = (x_1, x_2, \ldots, x_n) \text{ s.t. } x_i = -1 \text{ if } i \in X \text{ and } x_i = 1 \text{ otherwise}$$

That is, the sign of $x$ will determine if a node is "inside" or "outside".

Review: eigenvalues and eigenvectors

$$L = \begin{pmatrix} L_{11} & L_{12} & \cdots & L_{1n} \\ L_{21} & L_{22} & \cdots & L_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n1} & L_{n2} & \cdots & L_{nn} \end{pmatrix}$$

If $k = \lambda x$ then $\lambda$ is an eigenvalue of $L$.

$x$ is an eigenvector of $L$ with a corresponding eigenvalue $\lambda$.

Note: that $Dx$ is also an eigenvector, for any constant $D$.

So we have

$$x^T L x = \left( x_1, \ldots, x_n \right) \begin{pmatrix} L_{11} & \cdots & L_{1n} \\ \vdots & \ddots & \vdots \\ L_{n1} & \cdots & L_{nn} \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$= \sum_{i=1}^{n} x_i L_{ii} + \sum_{i<n} x_i L_{ij}$$

$$= \sum_{i=1}^{n} x_i L_{ii}$$

$x$ which minimizes this (which is the same as the maximization) will also give us the separation between inside and outside.

Spectral Based Partitioning

Solving the generalized eigensystem

$$Lx = \lambda Dx$$

And taking $x$ for the smallest eigenvalue yields the solution.
Spectral Based Partitioning

Recall: The pole matrix is positive definite.

Typically spectral based partitioning involves positive indefinite matrices, so that the smallest eigenvalue is 0, corresponding to x with all the same sign. This paper allows one to use the smallest eigenvalue, not second smallest because pole matrix is positive definite, because of $L_{ii} > 0$. 