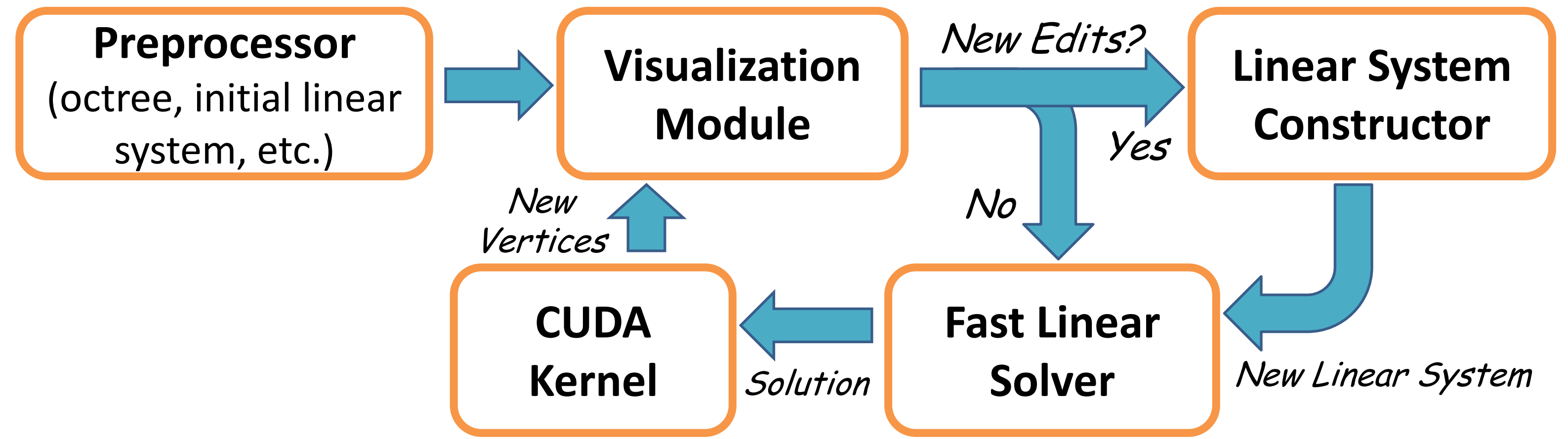




A Real-Time Screened-Poisson Solver for Interactive Surface Editing

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Abstract
We present a real-time system for editing large 3D meshes. The system supports local modulation of surface detail, expressing the position of vertices on the edited geometry as the solution to a linear system defined over the surface. The interactivity of our system is enabled by the design of an efficient sparse linear solver, providing an interface through which users can explore a broad landscape of possible surface modifications.



Motivation
Geometry-Processing: Drawing on the analogy between the eigenvectors of the mesh Laplacian and the Fourier basis, Taubin [1995] describes geometry processing as convolution:

$$\text{Geometry}_{new}(p) = \text{Geometry}_{old}(p) * H(p)$$

Image-Processing: Bhat *et al.* [2006] formulate a class of image filters as optimizations seeking to preserve the pixel values while amplifying or dampening the gradients:

$$\alpha \|\text{Image}_{new} - \text{Image}_{old}\|^2 + \|\nabla \text{Image}_{new} - \beta \nabla \text{Image}_{old}\|^2$$

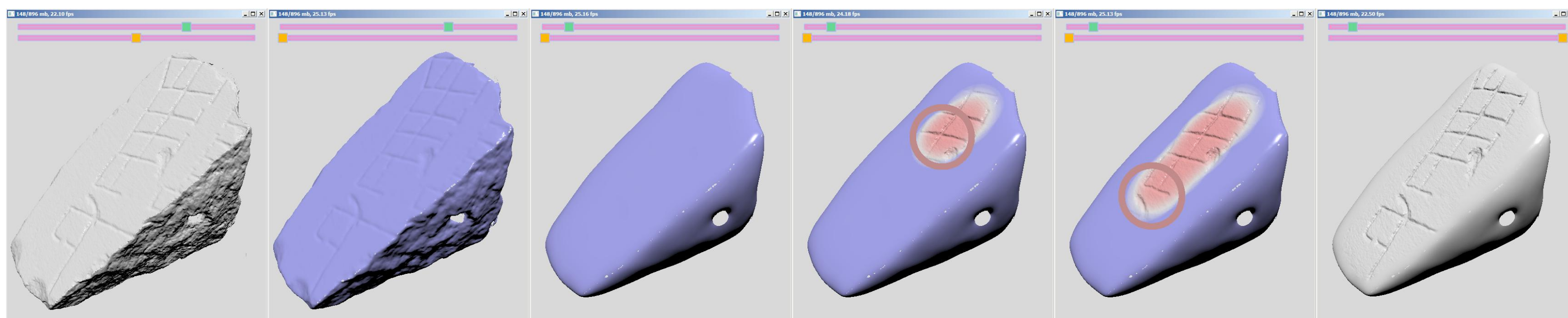
Error is minimized by solving a *screened-Poisson* system:

$$(\alpha I - \Delta)\text{Image}_{new} = (\alpha - \beta \Delta)\text{Image}_{old}$$

which is equivalent to convolution with the filter

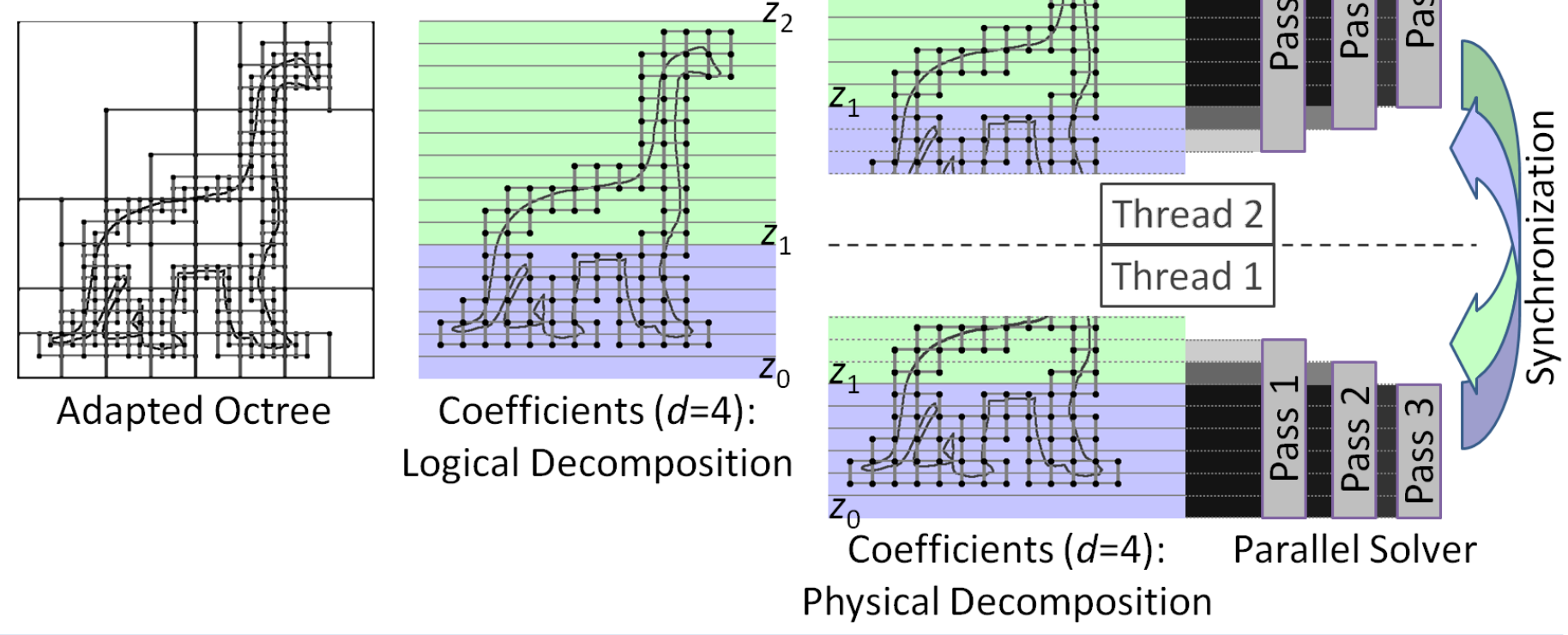
$$\hat{H}_{\alpha,\beta}(\omega) = \frac{\alpha + \beta \omega^2}{\alpha + \omega^2}$$

Approach
We extend the results of Bhat *et al.* to geometry-processing, solving a screened-Poisson equation to implement the convolution described by Taubin. We do this by using the coordinate functions to define the constraints and replacing the Euclidean Laplacian with the Laplace-Beltrami operator.



Advantages
➤ Solving the screened-Poisson system, we obtain the effects described by Taubin, without having to solve an eigenvalue problem.
➤ Formulating the convolution as a solution to a linear system, we bypass the stationarity constraints, allowing us to assign variable gradient weights for local editing.

Implementation Features
➤ Octree-based finite-element system
➤ Parallelization via domain decomposition
➤ Cache-friendly access via temporal blocking
➤ CUDA kernel for coordinate evaluation
➤ Per-frame solution relaxation



Related Work
Taubin, G. 1995. A signal processing approach to fair surface design. In *ACM SIGGRAPH Conference Proceedings*, 351–358.
Bhat, P., Curless, B., Cohen, M., and Zitnick, L. 2008. Fourier analysis of the 2D screened Poisson equation for gradient domain problems. In *Proceedings of the 10th European Conference on Computer Vision*, 114–128.

Results
We develop a system that can process meshes consisting of 1,000,000 vertices at over 25 fps.