1 Min-cost bounded path (50 points)

Suppose are are given a directed acyclic graph $G = (V, E)$, a cost function $c : E \to \mathbb{N}$, a length function $\ell : E \to \mathbb{N}$, two nodes $s, t \in V$, and a length bound $L$. Our goal is to compute the minimum cost $s \to t$ path whose length is at most $L$. In other words, find the path $P$ from $s$ to $t$ with $\sum_{e \in P} \ell(e) \leq L$ which minimizes $\sum_{e \in P} c(e)$.

(a) (25 points) Give a quasipolynomial-time exact algorithm for this problem (i.e., an algorithm which solves the problem but runs in time polynomial in $n, m$ and $\max_{e \in E} c(e), \max_{e \in E} \ell(e)$).

(b) (25 points) Using your algorithm from the previous part, give an FPTAS (a $(1 + \varepsilon)$-approximation which runs in time polynomial in the size of the instance and $1/\varepsilon$, i.e., polynomial in $n, m, \log(\max_{e \in E} c(e)), \log(\max_{e \in E} \ell(e)),$ and $1/\varepsilon$).

2 Min-degree Steiner Tree (50 points)

The Min-degree Steiner tree problem is the same as the Steiner tree problem, except instead of minimizing total cost our goal is to minimize the maximum degree. More formally, we are given an undirected graph $G = (V, E)$ and a subset $D \subseteq V$ of terminals. The goal is to find a tree $T$ in $G$ which spans all of $D$ (but not necessarily all of $V$) and which minimizes the maximum degree.

Show how to modify the local search algorithm for min-degree spanning tree to get a local search algorithm for min-degree Steiner tree that runs in polynomial time and returns a tree with maximum degree at most $2\Delta^* + \lceil \log n \rceil$ (where $\Delta^*$ is the maximum degree of the optimal tree).

Hint: you might need to slightly redefine a $u$-improvement, since adding a single edge might not create a fundamental cycle anymore. What kind of structure “acts” in a way equivalent to a non-tree edge in spanning trees?