601.436/636 Algorithmic Game Theory **Spring 2020**
Homework #5 **Due:** April 30, 2020, 3pm

Remember: you may work in groups of up to three people, but must write up your solution entirely on your own. Collaboration is limited to discussing the problems – you may not look at, compare, reuse, etc. any text from anyone else in the class. Please include your list of collaborators on the first page of your submission. You may use the internet to look up formulas, definitions, etc., but may not simply look up the answers online.

Please include proofs with all of your answers, unless stated otherwise. Your solution must by typeset (*not* handwritten), and must be submitted by gradescope.

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1 Auctions with Distinct Goods (33 points)

Consider an auction setting with a set M of distinct goods. Each bidder i has a publicly known subset $T_i \subseteq M$ of goods that it wants, and a private valuation v_i of getting them. An allocation is a partition $(A_1, A_2, \ldots, A_n)$ of the goods (or, equivalently, a function from the goods to the bidders). The *social surplus* of an allocation is the sum over all bidders who receive their desired items of their valuations: if $(A_1, A_2, \ldots, A_n)$ is an allocation then the social surplus is $\sum_{i:T_i \subseteq A_i} v_i$.

(a) (11 points) Prove that this is a single-parameter environment.

(b) (11 points) Here is a natural greedy allocation rule, given a reported bid b_i from each player i:

  (a) Initialize $S = \emptyset$, $X = M$.
  (b) Sort and re-index the bidders so that $b_1 \geq b_2 \geq \cdots \geq b_n$.
  (c) For $i = 1, 2, 3, \ldots, n$:
    - If $T_i \subseteq X$, then:
      – Delete T_i from X.
      – Add i to S.
  (d) Return S (give the bidders in S their desired items)

  Does this algorithm define a monotone allocation rule? Prove it or give an explicit counterexample.

(c) (11 points) Prove that if all bidders report truthfully and have sets T_i of cardinality at most d, then the outcome of the allocation rule in (b) has social surplus at least $1/d$ times the social surplus of the optimal (surplus-maximizing) allocation.

2 Unit-Demand Valuations (33 points)

Consider a combinatorial auction with n players and item set M with $m = |M|$. A player i has a *unit-demand* valuation if there exist parameters $v_i^1, v_i^2, \ldots, v_i^m \in \mathbb{R}_{\geq 0}$ (one parameter per item) such that $v_i(S) = \max_{j \in S} v_i^j$ for all $S \subseteq M$ (and $v_i(\emptyset) = 0$). In other words, the value of a bundle for player i is determined by the single most valuable element in that bundle (from the perspective of player i).

Give a mechanism for combinatorial auctions in which all players have unit-demand valuations which satisfies the following properties:

(a) It is incentive-compatible,

(b) It maximizes social welfare (i.e., maximizes $\sum_{i=1}^{n} v_i(S_i)$ where S_i is the bundle given to player i by the mechanism), and

(c) It runs in time polynomial in n and m.

Hint: Show that VCG can be implemented in polynomial time (in this setting) by reducing it to a well-known graph algorithm problem.

3 VCG with Pseudonyms (34 points)

Consider a combinatorial auction in which players can submit multiple bids under different names, unbeknownst to the mechanism. The allocation and payment of a player is the union and sum of the allocations and payments, respectively, assigned to all of his/her pseudonyms.

(a) (17 points) Exhibit a combinatorial auction and player valuations such that, in the VCG mechanism, there is a player who can earn higher utility by submitting multiple bids under pseudonyms than by bidding truthfully as a single agent (assuming others bid truthfully).

(b) (17 points) Can this ever happen in a second-price single-item auction?