Biometric Authentication Revisited:
Understanding the Impact of Wolves in Sheep's Clothing

Lucas Ballard
Department of Computer Science
Johns Hopkins University

Fabian Monrose
Department of Computer Science
Johns Hopkins University

Daniel Lopresti
Department of Computer Science & Engineering
Lehigh University

Abstract

Biometric security is a topic of rapidly growing importance, especially as it applies to user authentication and key generation. In this paper, we describe our initial steps towards developing evaluation methodologies for behavioral biometrics that take into account threat models which have largely been ignored. We argue that the pervasive assumption that forgers are minimally motivated (or, even worse, naïve), or that attacks can only be mounted through manual effort, is too optimistic and even dangerous. To illustrate our point, we analyze a handwriting-based key-generation system and show that the standard approach of evaluation significantly overestimates its security. Additionally, to overcome current labor-intensive hurdles in performing more accurate assessments of system security, we present a *generative attack* model based on concatenative synthesis that can provide a rapid indication of the security afforded by the system. We show that our generative attacks match or exceed the effectiveness of forgeries rendered by the skilled humans we have encountered.

1 Introduction

The security of many systems relies on obtaining human input that is assumed to not be readily reproducible by an attacker. Passwords are the most common example, though the assumption that these are not reproducible is sensitive to the number of guesses that an attacker is allowed. In *online* attacks, the adversary must submit each request to a nonbypassable reference monitor (e.g., a login prompt) that accepts or declines the password and permits a limited number of incorrect attempts. In contrast, an *offline* attack permits the attacker to make a number of guesses at the password that is limited only by the resources available to the attacker, i.e., time and memory. When passwords are used to derive cryptographic keys, they are susceptible to offline attacks.

An alternative form of user input that is intended to be difficult for attackers to reproduce are biometrics. Like passwords, biometrics have typically been used as a technique for a user to authenticate herself to a reference monitor that can become unresponsive after a certain number of failed attempts. However, biometrics also have been explored as a means for generating user-specific cryptographic keys (see for example, [30, 21]). As with password-generated keys, there is insufficient evidence that keys generated from biometric features alone will typically survive offline attacks. As such, an alternative that we and others have previously explored is *password hardening* whereby a cryptographic key is generated from both a password and dynamic biometric features of the user while entering it [22, 23].

While these directions may indeed allow for the use of biometrics in a host of applications, we believe the manner in which biometric systems have been tested in the literature (including our prior work) raises some concerns. In particular, this work demonstrates the need for adopting more realistic adversarial models when performing security analyses. Indeed, as we show later, the impact of forgeries generated under such conditions helps us to better understand the security of certain biometric-based schemes.

Our motivation for performing this analysis is primarily to show that there exists a disconnect between realistic threats and typical "best" practices [17] for reporting biometric performance—one that requires rethinking as both industry and the research community gains momentum in the exploration of biometric technologies. We believe that the type of analysis presented herein is of primary importance for the use of biometrics for authentication and cryptographic key generation (e.g., [21, 7, 2, 12]), where *weakest-link* analysis is paramount.

Moreover, to raise awareness of this shortcoming we explore a particular methodology in which we assume that the adversary utilizes indirect knowledge of the target user's biometric features. That is, we presume that

the attacker has observed measurements of the biometric in contexts outside its use for security. For example, if the biometric is the user's handwriting dynamics generated while providing input via a stylus, then we presume the attacker has samples of the user's handwriting in another context, captured hardcopies of the user's writing, or writings from users of a similar style. We argue that doing so is more reflective of the real threats to biometric security. In this paper, we explore how an attacker can use such data to build *generative models* that predict how a user would, in this case, write a text, and evaluate the significance of this to biometric authentication.

2 Biometric Authentication

Despite the diversity of approaches examined by the biometrics community [1], from the standpoint of this investigation several key points remain relatively constant. For instance, the traditional procedure for applying a biometric as an authentication paradigm involves sampling an input from a user, extracting an appropriate set of features, and comparing these to previously stored templates to confirm or deny the claimed identity. While a wide range of features have been investigated, it is universally true that system designers seek features that exhibit large inter-class variability and small intra-class variability. In other words, two different users should be unlikely to generate the same input features, while a single user ought to be able to reproduce her own features accurately and repeatably.

Likewise, the evaluation of most biometric systems usually follows a standard model: enroll some number of users by collecting training samples, e.g., of their handwriting or speech. At a later time, test the rate at which users' attempts to recreate the biometric to within a predetermined tolerance fail. This failure rate is denoted as the False Reject Rate (FRR). Additionally, evaluation usually involves assessing the rate at which one user's input (i.e., an impostor) is able to fool the system when presented as coming from another user (i.e., the target). This evaluation yields the False Accept Rate (FAR) for the system under consideration. A tolerance setting to account for natural human variation is also vital in assessing the limits within which a sample will be consider as genuine, while at the same time, balancing the delicate trade-off of resistance to forgeries. Typically, one uses the equal error rate (EER)—that is, the point at which the FRR and the FAR are equal—to describe the accuracy of a given biometric system. Essentially, the lower the EER, the higher the accuracy.

Researchers also commonly distinguish between forgeries that were never intended to defeat the system ("random" or naïve forgeries), and those created by a user who was instructed to make such an attempt given infor-mation about the targeted input (i.e., so-called "skilled" forgeries). However, the evaluation of biometrics under such weak security assumptions can be misleading. Indeed, it may even be argued that because there is no strong means by which one can define a good forger and prove her existence (or non-existence), such analysis is theoretically impossible [29]. Nevertheless, the biometric community continues to rely on relatively simple measures of adversarial strength, and most studies to date only incorporate unskilled adversaries, and very rarely, "skilled" impersonators [13, 29, 11, 19, 20, 15].

This general practice is troubling as the evaluation of the FAR is likely to be significantly underestimated [29, 28]. Moreover, we believe that this relatively ad hoc approach to evaluation misses a significant threat: the use of *generative models* to create synthetic forgeries which can form the basis for sophisticated *automated* attacks on biometric security. This observation was recently reiterated in [32], where the authors conjectured that although the complexity of successful impersonations on various biometric modalities can be made formidable, biometric-based systems might be defeated using various strategies (see for example [31, 9, 26, 15]). As we show later, even rather simplistic attacks launched by successive replication of synthetic or actual samples from a representative population can have adverse effects on the FAR—particularly for the weakest users (i.e., the so-called "Lambs" in the biometric jargon for a hypothetical menagerie of users [3]).

In what follows, we provide what we believe is the most in-depth study to date that emphasizes the extent of this problem. Furthermore, as a first step towards providing system evaluators with a stronger methodology for quantifying performance under various threats, we describe our work on developing a prototype toolkit using handwriting dynamics as a case in point.

3 Handwriting Biometrics

Research on user authentication via handwriting has had a long, rich history, with hundreds of papers written on the topic. The majority of this work to date has focused on the problem of signature verification [27]. Signatures have some well known advantages: they are a natural and familiar way of confirming identity, have already achieved acceptance for legal purposes, and their capture is less invasive than most other biometric schemes [6]. While each individual has only one true signature—a notable limitation—handwriting in general contains numerous idiosyncrasies that might allow a writer to be identified.

In considering the mathematical features that can be extracted from the incoming signal to perform authentication, it is important to distinguish between two dif-

ferent classes of inputs. Data captured by sampling the position of a stylus tip over time on a digitizing tablet or pen computer are referred to as *online* handwriting, whereas inputs that are presented in the form of a 2-D bitmap (e.g., scanned off of a piece of paper) are referred to as *offline* handwriting. To avoid confusion with the traditional attack models in the security community, later on in this paper we shall eschew that terminology and refer to the former as covering both temporal and spatial information, whereas the latter only covers spatial information. Features extracted from offline handwriting samples include bounding boxes and aspect ratios, stroke densities in a particular region, curvature measurements, etc. In the online case, these features are also available and, in addition, timing and stroke order information that allows the computation of pen-tip velocities, accelerations, etc. Studies on signature verification and the related topic of handwriting recognition often make use of 50 or more features and, indeed, feature selection is itself a topic for research. The features we use in our own work are representative of those commonly reported in the field [8, 33, 18, 14]. Repeatability of features over time is, of course, a key issue, and it has been found that dynamic and static features are equally repeatable [8].

In the literature, performance figures (i.e., EER) typically range from 2% to 10% (or higher), but are difficult to compare directly as the sample sizes are often small and test conditions quite dissimilar [5]. Unfortunately, forgers are rarely employed in such studies and, when they are, there is usually no indication of their proficiency. Attempts to model attackers with a minimal degree of knowledge have involved showing a static image of the target signature and asking the impostor to try to recreate the dynamics [24]. The only serious attempt we are aware of, previous to our own, to provide a tool for training forgers to explore the limits of their abilities is the work by Zoebisch and Vielhauer [35]. In a small preliminary study involving four users, they found that showing an image of the target signature increased false accepts, and showing a dynamic replay doubled the susceptibility to forgeries yet again. However, since the verification algorithm used was simplistic and they do not report false reject rates, it is difficult to draw more general conclusions.

To overcome the "one-signature-per-user" (and hence, one key) restriction, we employ more general passphrases in our research. While signatures are likely to be more user-specific than arbitrary handwriting, results from the field of forensic analysis demonstrate that writer identification from a relatively small sample set is feasible [10]. Indeed, since this field focuses on handwriting extracted from scanned page images, the problem we face is less challenging in some sense since we have access to dynamic features in addition

to static. Another concern, user habituation [5], is addressed by giving each test subject enough time to become comfortable with the experimental set-up and requiring practice writing before the real samples are collected. Still, this is an issue and the repeatability of non-signature passphrases is a topic for future research.

4 Experimental Design

We collected data over a two month period to analyze six different forgery styles. We consider three standard evaluation metrics: *naïve*, *static*, and *dynamic* [1] forgeries [13, 29, 11], as well as three metrics that will provide a more realistic definition of security: *naïve**, *trained*, and *generative*. Naïve, or "accidental", forgeries are not really forgeries in the traditional sense; they are measured by authenticating one user's natural writing samples of a passphrase against another user's template for the same passphrase. Static (resp. dynamic) forgeries are created by humans after seeing static (resp. real-time) renderings of a target user's passphrase. Naïve* forgeries are similar to naïve forgeries except that only writings from users of a similar style are authenticated against a target user's template. Trained forgeries are generated by humans under certain conditions, which will be described in greater detail later. Lastly, generative forgeries exploit information about a target user to algorithmically generate forgeries. Such information may include samples of the user's writing from a different context or general population statistics.

4.1 Data Collection

Our results are based on 11,038 handwriting samples collected on digitized pen tablet computers from 50 users during several rounds. We used NEC VersaLite Pad and HP Compaq TC1100 tablets as our writing platforms. The specifics of each round will be addressed shortly. To ensure that the participants were well motivated and provided writing samples reflective of their natural writing (as well as forgery attempts indicative of their innate abilities), several incentives were awarded for the most consistent writers, the best/most dedicated forgers, etc.

Data collection was spread across three rounds. In round I, we collected two distinct data sets. The first set established a baseline of "typical" user writing. After habituation on the writing device [5], users were asked to write five different phrases, consisting of two-word oxymorons, ten times each. We chose these phrases as they were easy to remember (and therefore, can be written naturally) and could be considered of reasonable length. Signatures were not used due to privacy concerns and strict restrictions on research involving human-subjects.

More importantly, in the context of key-generation, signatures are not a good choice for a hand-writing biometric as the compromise of keying material could prevent a user from using the system thereafter. This part of the data set was used for two purposes: to establish biometric templates to be used for authentication, and to provide samples for naive and naive* forgeries. To create a strong underlying representative system, users were given instructions to write as naturally (and consistently) as possible.

The second data set from `round I`, our "generative corpus", was used to create our generative forgeries and consisted of a set of 65 oxymorons. This set was restricted so that it did not contain any of the five phrases from the first data set, yet provided coverage of the first set at the bi-gram level. As before, we chose oxymorons that were easy to recall, and users were asked to write one instance of each phrase as naturally as possible. The average elapsed time for `round I` was approximately one hour.

`Round II` started approximately two weeks later. The same set of users wrote the five phrases from `round I` ten times. Additionally, the users were asked to forge representative samples (based on writing style, handedness of the original writer, and gender) from `round I` to create two sets of 17 forgeries. First, users were required to forge samples after seeing *only* a static representation. This data was used for our static forgeries. Next, users were asked to forge the same phrases again, but this time, upon seeing a real-time rendering of the phrase. At this stage, the users were instructed to make use of the real-time presentation to improve their rendering of the spatial features (for example, to distinguish between one continuous stroke versus two strokes that overlap) and to replicate the temporal features of the writing. This data comprised our dynamic forgeries. On average, `round II` took approximately 90 minutes for each user.

Lastly, in `round III` we selected nine users from `round II` who, when evaluated using the authentication system to be described in §4.2 and §4.3, exhibited a natural tendency to produce better forgeries than the average user in our study (although we did not include all of the best forgers). This group consisted of three "skilled" (but untrained) forgers for each writing style. (One of "cursive", "mixed", or "block", where the classification is based on the percent of the time that users connect adjacent characters.) Each skilled forger was asked to forge writing from the style which they exhibited an innate ability to replicate and was provided with a general overview and examples of the types of temporal and spatial characteristics that handwriting systems typically capture. As we were trying to examine (and develop) truly skilled adversaries, our forgers were asked to forge

15 writing samples from their specified writing style, with 60% of the samples coming from the weakest 10 targets, and the other 40% chosen at random. (In §5 we also provide the results of our trained forgeries against the entire population.) From this point on, these forgers (and their forgeries) will be referred to as "trained" forgers. We believe that the selection of the naturally skilled forgers, the additional training, and the selection of specific targets produced adversaries who realistically reflect a threat to biometric security.

The experimental setup for these educated forgers is as follows. First, a real-time reproduction of the target sample is displayed (at the top half of the tablet) and the forger is allowed to attempt forgeries (at her own pace) with the option of saving the attempts she liked. She can also select and replay her forgeries and compare them to the target. In this way, she is able to fine-tune her attempts by comparing the two writing samples. Next, she selects the forgery she believes to be her best attempt, and proceeds to the next target. The average elapsed time for this round was approximately two hours.

4.2 Authentication System

In order to have a concrete platform to measure the `FAR` for each of our six forgery styles, we loosely adapted the system presented in [34, 33] for generation of "biometric hashes". We note that our results are system-independent as we are only evaluating biometric *inputs*, for which we evaluated features that are reflective of the state of the art [14, 18, 8, 33].

For completeness, we briefly describe relevant aspects of the system; for a more detailed description see [33]. To input a sample to the system, a human writes a passphrase on an electronic tablet. The sample is represented as three signals parameterized by time. The discrete signals $x(t)$ and $y(t)$ specify the location of the pen on the writing surface at time t, and the binary signal $p(t)$ specifies whether the pen is up or down at time t. The tablet computes a set of n statistical features $(f_1, \ldots, f_n)$ over these signals. These features comprise the actual input to the authentication or key-generation system.

During an enrollment phase, each legitimate user writes a passphrase a pre-specified number (m) of times, and the feature values for each sample are saved. Let $f_{i,1}, \ldots, f_{i,n}$ denote the feature values for sample i. Using the feature values from each user and passphrase, the system computes a global set of tolerance values ($T = \{\epsilon_1, \ldots, \epsilon_n\}$) to be used to account for natural human variation [34]. Once the m readings have been captured, a biometric template is generated for each user and passphrase as follows: Let $\ell'_j = \min_{i \in [1,m]} f_{i,j}$, $h'_j = \max_{i \in [1,m]} f_{i,j}$, and $\Delta_j = h'_j - \ell'_j + 1$. Set $\ell_j = \ell'_j - \Delta_j \epsilon_j$, and $h_j = h'_j + \Delta_j \epsilon_j$. The resulting template

is an $n \times 2$ matrix of values $\{\{\ell_1, h_1\}, \ldots, \{\ell_n, h_n\}\}$.

Later, when a user provides a sample with feature values $f_1, \ldots, f_n$ for authentication, the system checks whether $f_j \in [\ell_j, h_j]$ for each feature f_j. Each $f_j \notin [\ell_j, h_j]$ is deemed an error, and depending on the threshold of errors tolerated by the system, the attempt is either accepted or denied. We note that as defined here, templates are insecure because they leak information about a user's feature values. We omit discussion of securely representing biometric templates (see for example [22, 4]) as this is not a primary concern of this research.

4.3 Feature Analysis

Clearly, the security of any biometric system is directly related to the quality of the underlying features. A detailed analysis of proposed features for handwriting verification is presented in [33], although we argue that the security model of that work sufficiently differs from our own and so we believe a new feature-evaluation metric was required. In that work, the quality of a feature was measured by the deviation of the feature and entropy of the feature across the population. For our purposes, these evaluation metrics are not ideal: we are not only concerned with the entropy of each feature, but rather how difficult the feature is to *forge* — which we argue is a more important criteria. When systems are evaluated using purely naïve forgeries, then entropy could be an acceptable metric. However, as we show later, evaluation under naïve forgeries is not appropriate [2].

As our main goal is to highlight limitations in current practices, we needed to evaluate a robust yet usable system based on a strong feature set. To this end, we implemented 144 state of the art features [33, 8, 25, 14] and evaluated each based on a quality metric (Q) defined as follows. For each feature f, we compute the proportion of times that f was missed by legitimate users in our study, denoted r_f, and the proportion of times that f was missed by forgers from round II (with access to dynamic information), denoted a_f. Then, $Q(f) = (a_f - r_f + 1)/2$, and the range of Q is $[0, 1]$. Intuitively, features with a quality score of 0 are completely useless—i.e., they are *never* reliably reproduced by original users and are *always* reproduced by forgers. On the other hand, features with scores closer to 1 are highly desirable when implementing biometric authentication systems.

For our evaluation, we divided our feature set into two groups covering the temporal and spatial features, and ordered each according to the quality score. We then chose the top 40 from each group, and disregarded any with a FRR greater than 10%. Finally, we discounted any features that could be inferred from others (e.g., given the

width and height of a passphrase as rendered by a user, then a feature representing the ratio between width and height is redundant). This analysis resulted in what we deem the 36 best features—15 spatial and 21 temporal—described in Appendix A.

5 Human Evaluation

This section presents the results for the five evaluation metrics that use forgeries generated by humans. Before we computed the FRR and the FAR, we removed the outliers that are inherent to biometric systems. For each user, we removed all samples that had more than $\delta = 3$ features that fell outside $k = 2$ standard deviations from that user's mean feature value. The parameters δ and k were empirically derived. We also did not include any samples from users (the so-called "Goats" [3]) who had more than 25% of their samples classified as outliers. Such users "Failed to Enroll" [17]; the FTE rate was $\approx 8.7\%$. After combining this with outlier removal, we still had access to 79.2% of the original data set.

To compute the FRR and the FAR we use the system described in §4.2 using the features from §4.3. The FRR is computed as follows: we repeatedly randomly partition a user's samples into two groups and use the first group to build a template and authenticate the samples in the second group against the template. To compute the FAR we use all of the user's samples to generate a template and then authenticate the forgeries against this template.

5.1 Grooming Sheep into Wolves

Our experiments were designed to illustrate the discrepancy in perceived security when considering traditional forgery paradigms and a more stringent, but realistic, security model. In particular, we assume that at the very minimum, that a realistic adversary (1) attacks victims who have a writing style that the forger has a natural ability to replicate, (2) has knowledge of how biometric authentication systems operate, and (3) has a vested interest in accessing the system, and therefore is willing to devote significant effort towards these ends.

Figure 1 presents ROC curves for forgeries from impersonators with varying levels of knowledge. The plot denoted FAR-naïve depicts results for the traditional case of naïve forgeries widely used in the literature [13, 29, 11]. In these cases, the impersonation attempts simply reflect taking one user's natural rendering of phrase p as an impersonation attempt on the target writing p. Therefore, in addition to ignoring the target writer's attributes as is naturally expected of forgers, this classification makes no differentiation based on the

forger's or the victim's style of writing, and so may include, for example, block writers "forging" cursive writers. Arguably, such forgeries may not do as well as the less standard (but more reasonable) naïve* classification (FAR-naïve*) where one only attempts to authenticate samples from writers of similar styles.
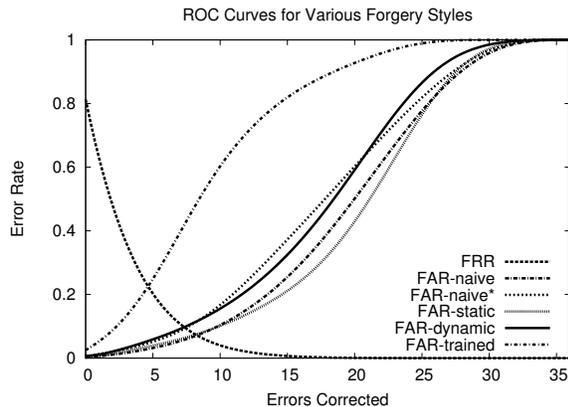

Figure 1: Overall ROC curves for naïve, naïve*, static, dynamic, and trained forgers.
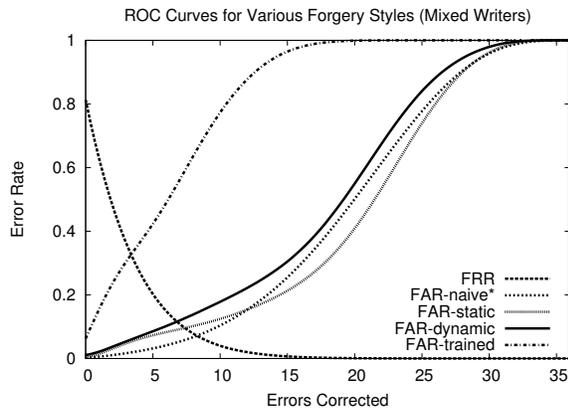

Figure 2: ROC curves against all *mixed* writers. This grouping appeared the easiest to forge by the users in our study.

The FAR-static plots represent the success rate of forgers who receive access to only a static rendering of the passphrase. By contrast, FAR-dynamic forgeries are produced after seeing (possibly many) real-time renderings of the image. One can easily consider this a realistic threat if we assume that a motivated adversary may capture the writing on camera, or more likely, may have access to data written electronically in another context. Lastly, FAR-trained presents the resulting success rate

of forgeries derived under our forgery model which captures a more worthy opponent. Notice that when classified by writing style, the trained forgers were very successful against mixed writers (Figure 2).

Intuitively, one would expect that forgers with access to dynamic and/or static representations of the target writing should be able to produce better forgeries than those produced under the naïve* classification. This is not necessarily the case, as we see in Figure 1 that at some points, the naïve* forgeries do better than the forgeries generated by forgers who have access to static and/or dynamic information. This is primarily due to the fact that the naïve* classification reflects users' normal writing (as there is really no forgery attempt here). The natural tendencies exhibited in their writings appear to produce better "forgeries" than that of static or dynamic forgers (beyond some point), who may suffer from unnatural writing characteristics as a result of focusing on the act of forging.

One of the most striking results depicted in the figures is the significant discrepancy in the FAR between standard evaluation methodologies and that of the trained forgeries captured under our strengthened model. While it is tempting to directly compare the results under the new model to those under the more traditional metrics (i.e., by contrasting the FAR-trained error rate at the EER under one of the older models), such a comparison is *not* valid. This is because the forgers under the new model were more knowledgeable with respect to the intricacies of handwriting verification and had performed style-targeted forgeries.

However, the correct comparison considers the EERs under the two models. For instance, the EER for this system under FAR-trained forgeries is approximately 20.6% at four error corrections. However, for the more traditional metrics, one would arrive at EERs of 7.9%, 6.0%, 5.5% under evaluations of dynamic, static and naïve forgeries, respectively. These results are indeed inline with the current state of the art [13, 29, 11]. Even worse, under the most widely used form of adversary considered in the literature (i.e., naïve) we see that the security of this system would be over-estimated by nearly 375%!

Forger Improvement Figure 3 should provide assurance that the increase in forgery quality is not simply a function of selecting naturally skilled individuals from round II to participate in round III. The graph shows the improvement in FAR between rounds II and III for the trained forgers. We see that the improvement is significant, especially for the forgers who focused on mixed and block writers. Notice that at the EER (at seven errors) induced by forgers with access to dynamic information (Figure 1), our trained mixed, block, and cursive forgers improved their FAR by 0.47, 0.34, and 0.18, re-
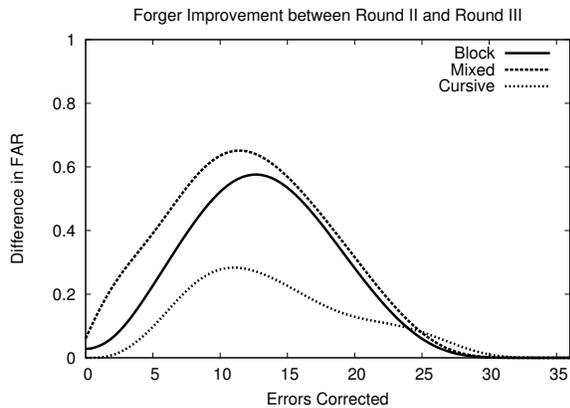
Figure 3: Forger improvement between `rounds II` and `III`.

spectively. This improvement results from less than two hours of training and effort, which is likely much less than what would be exerted by a dedicated or truly skilled forger.

The observant reader will note that the trained forgers faced a different distribution of "easy" targets in Round `III` then they did in Round `II`. We did this to analyze the system at its weakest link. However, after normalizing the results so that both rounds had the same makeup of "easy" targets, the change in `EER` is statistically insignificant, shifting from 20.6% to 20.0% at four errors corrected.

6 Generative Evaluation

Unfortunately, finding and training "skilled" forgers is a time (and resource) consuming endeavor. To confront the obstacles posed by wide-scale data collection and training of good impersonators, we decided to explore the use of an automated approach using generative models as a supplementary technique for evaluating behavioral biometrics. We investigated whether an automated approach, using limited writing samples from the target, could match the false accept rates observed for our trained forgers in §5.1. We believe that such generative attacks themselves may be a far more dangerous threat that, until now, have yet to be studied in sufficient detail.

For the remaining discussion we explore a set of threats that stem from generative attacks which assume knowledge that spans the following spectrum:

I. *General population statistics*: Gleaned, for example, via the open sharing of test data sets by the research community, or by recruiting colleagues to provide writing samples.

II. *Statistics specific to a demographic of the targeted user*: In the case of handwriting, we assume the attacker can extract statistics from a corpus collected from other users of a similar writing style (e.g., cursive).

III. *Data gathered from the targeted user*: Excluding direct capture of the secret itself, one can imagine the attacker capturing copies of a user's handwriting, either through discarded documents or by stealing a PDA.

To make this approach feasible, we also explore the impact of these varying threats. A key issue that we consider is the amount of recordings one needs to make these scenarios viable attack vectors. As we show later, the amount of data required may be surprisingly small for the case of authentication systems based on handwriting dynamics.

6.1 A generative toolkit for performance testing

The approach to synthesizing handwriting we explore here is to assemble a collection of basic units (n-grams) that can be combined in a concatenative fashion to mimic authentic handwriting. In this case, we do not make use of an underlying model of human physiology, rather, creation of the writing sample is accomplished by choosing appropriate n-grams from an inventory that may cover writing from the target user (scenario III above) as well as representative writings by other members of the population at large (scenarios I and II). The technique we apply here expands upon earlier rudimentary work [16], and is similar in flavor to approaches taken to generate synthesized speech [21] and for text-to-handwriting conversion [9].

6.1.1 Forgeries

As noted earlier, each writing sample consists of three signals parameterized by time: $x(t)$, $y(t)$ and $p(t)$. The goal of our generative algorithm is to generate t, $x(t)$, $y(t)$ and $p(t)$ such that the sample is not only accepted as authentic, but relies on acquiring a minimal amount of information from the target user (again, in a different security context). In particular, when attacking user u, we assume the adversary has access to a generative corpus $\mathcal{G}_u$, in addition to samples from users of similar writing styles $\mathcal{G}_S$; where S is one of "block", "mixed", or "cursive". We assume that both $\mathcal{G}_u$ and $\mathcal{G}_S$ are annotated so that there is a bijective map between the characters of each phrase and the portion of the signal that represents each character. As is the case with traditional compu-

tations of the EER we also assume that passphrase p is known.

General Knowledge Assume that the adversary wishes to forge user u with passphrase p and writing style S. Ideally, she would like to do so using a minimal amount of information directly collected from u. Fortunately, the success of the naïve* forgeries from §5 suggests that a user's writing style yields a fair amount of pertinent information that can potentially be used to replicate that user's writing. Thus, to aid in generating accurate forgeries, the adversary can make use of several statistics computed from annotated writing samples in $\mathcal{G}_S \backslash \mathcal{G}_u$. In what follows, we discuss what turn out to be some very useful measures that can likely be easily generalized for other behavioral biometrics.

Denote as $P_c(i, j, c_1, c_2)$ the probability that writers of style S connect the ith stroke of c_1 to c_2, given that c_1 is comprised of j strokes. Let $P_c(i, j, c_1, *)$ be the probability that these writers connect the ith stroke of c_1 (again rendered with j strokes) to any adjacent letter. For example, many cursive writers will connect the first stroke of the letter 'i' to proceeding letters; for such writers $P_c(1, 2, \text{i}, *) \approx 1$. Note that in this case, the dot of the 'i' will be rendered after proceeding letters, we call this a "delayed" stroke.

Let $\delta_w(c_1, c_2)$ denote the median gap between the adjacent characters c_1 and c_2 (i.e., the distance between the maximum value of $x(t)$ for c_1 and the minimum value of $x(t)$ for c_2), $\delta_w(c_1, *)$ the median gap between c_1 and any proceeding character, and $\delta_w(*)$ the median gap between any two adjacent characters. Intuitively, $\delta_w(c_1, c_2) < 0$ if users tend to overlap characters. Similarly, let $\delta_t(c_1, c_2)$ denote the median time elapsed between the end of c_1 and the beginning of c_2. Definitions of $\delta_t(c_1, *)$ and $\delta_t(*)$ are analogous to those for δ_w.

Finally, the generative algorithm clearly must also make use of a user's pen-up velocity. This can be estimated from the population by computing the pen-up velocity for each element in $\mathcal{G}_S$ and using the 75th percentile of these velocities. We denote this value as v_S.

Having acquired her generalized knowledge, the adversary can now select and combine her choices of n-grams that will be used for concatenative-synthesis in the following manner:

n-gram Selection At a high level, the selection of n-grams that allow for a concatenative-style rendering of p involves a search of $\mathcal{G}_u$ for possible candidates. Let $\mathcal{G}_{u,p}$ be a set of u's renderings of various n-grams in p. There may be more than one element in $\mathcal{G}_{u,p}$ for each n-gram in p. The attacker selects k renderings $g_1, \ldots, g_k$ from $\mathcal{G}_{u,p}$ such that $g_1 || g_2 || \ldots || g_k = p$. Our selection algorithm is randomized, but biased towards longer n-grams.

However, the average length of each n-gram is small as shorter n-grams are required to "fill the gap" between larger n-grams. To explore the feasibility of our generative algorithm we ensure that g_i and g_{i+1} do not originate from the same writing sample, but an actual adversary might benefit from using n-grams from the same writing sample.

n-gram Combination Given the selection of n-grams $(g_1, \ldots, g_k)$ the attacker's task is to combine them to form a good representation of p. Namely, she must adjust the signals that compose each g_i (t_{g_i}, $x(t_{g_i})$, $y(t_{g_i})$ and $p(t_{g_i})$) to create a final set of signals that authenticates to the system. The algorithm is quite simple. At a high level, it proceeds as follows: The adversary normalizes the signals t_{g_i}, $x(t_{g_i})$ and $y(t_{g_i})$ by subtracting the respective minimum values from each element in the signal. The $y(t_{g_i})$ are shifted so that the baselines of the writing match across g_i. To finalize the spatial transforms, the adversary horizontally shifts each $x(t_{g_i})$ by

$$\delta_{x,i} = \delta_{x,i-1} + \max(x(t_{g_{i-1}})) + \delta_w(e_{i-1}, s_i)$$

where e_i (resp. s_i) is the last (resp. first) character in g_i and $\delta_{x,1} = 0$. Once the adversary has fixed the (x, y) coordinates, she needs to fabricate t and $p(t)$ signals to complete the forgery. Modifying $p(t)$ consists of deciding whether or not to connect adjacent n-grams. To do this, the adversary uses knowledge derived from the population. If e_{i-1} is rendered with j' strokes, and g_i starts with s_i, the adversary connects the jth stroke of e_{i-1} to s_i with probability $P_c(j, j', e_{i-1}, s_i)$. To generate a more realistic connection, the adversary smoothes the last points of e_{i-1} and the first points of s_i. Additionally, all strokes that occur after stroke j are "pushed" onto a stack, which is emptied on the next generated pen-up. This behavior simulates a true cursive writer returning to dot 'i's and cross 't's at the end of a word, processing characters closest to the end of the word first.

Adjusting the t signal is also straightforward. Let T be the time in $t_{g_{i-1}}$ that the last non-delayed stroke in e_{i-1} ends. If there are no delayed strokes in e_{i-1}, $T = \max(t_{g_{i-1}})$. Then, the adversary can simply shift t_{g_i}, $i > 1$ by

$$\delta_{\tau,i} = \delta_{\tau,i-1} + T + \delta_t(e_{i-1}, s_i)$$

and $\delta_{\tau,1} = 0$. The only other time shift occurs when delayed strokes are popped from the stack. We can make use of global knowledge to estimate the time delay by using v_S and the distance between the end of the previous stroke and the new stroke. Note that it is beneficial to take v_S as the 75th percentile instead of the median velocity because, for cursive writers in particular, the majority of pen-up velocities is dominated by the time between words. However, these velocities are intuitively

slower as the writer is now thinking about creating a new word as opposed to finishing a word that already exists.

If the adversary does not have access to the statistical measure $\delta_w(e_{i-1}, s_i)$, she can first base her estimate of inter-character spacing on $\delta_w(e_{i-1}, *)$, and then on $\delta_w(*)$. She proceeds similarly for the measures δ_t and P_c.

6.2 Results

To evaluate this concatenative approach we analyzed the quality of the generated forgeries on user u writing passphrase p. However, rather than using all 65 of the available samples from the generative corpus, we instead choose 15 samples at random from $\mathcal{G}_{u,p}$ — with the one restriction being that there must exist at least one instance of each character in p among the 15 samples. Recall that this generative corpus contains writing samples from u, but does not include p. The attacker's choice of n-grams $g_1, \ldots, g_k$ are selected from this restricted set.

Additionally, we limit $\mathcal{G}_S$ to contain only 15 randomly selected samples from each user with a similar writing style as u. Denote this set of writings as $\mathcal{G}'_S$. We purposefully chose to use small (and arguably, easily obtainable) data sets to illustrate the power of this concatenative attack. Our "general knowledge" statistics are computed from $\mathcal{G}'_S$. Example forgeries derived by this process are shown in Figure 4.
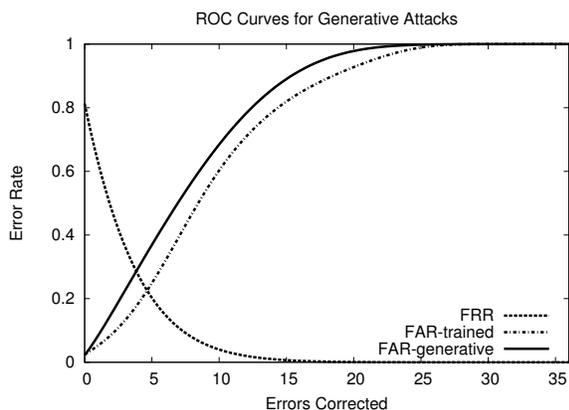

Figure 5: ROC curves for generative forgeries. Even with access to only limited information, the algorithm out-performs our trained forgers, shifting the EER from 20.6% at four errors to 27.4% at three errors.

We generated up to 25 forgery attempts for each user u and phrase p and used each as an attempt to authenticate to the biometric template corresponding to u under p. Figure 5 depicts the average FAR across all forgery attempts. As a baseline for comparison, we replot the

FRR and the FAR-trained plots from §5. The FAR-generative plot shows the results of the generative algorithm against the entire population. Observe that under these forgeries there is an EER of 27.4% at three error correction compared to an EER of 20.6% at four error corrections when considering our trained forgers.

We note that on average each generative attempt only used information from 6.67 of the target user's writing samples. Moreover, the average length of an n-gram was 1.64 characters (and was never greater than 4). More importantly, as we make no attempt to filter the output of the generative algorithm by rank-ordering the best forgeries, the results could be much improved. That said, we believe that given the limited information assumed here, the results of this generative attack on the security of the system warrant serious consideration. Furthermore, we believe that this attack is feasible because annotation of the samples in $\mathcal{G}_{u,p}$, while tedious, poses only a minor barrier to any determined adversary. For instance, in our case annotation was accomplished with the aide of an annotation tool that we implemented which is fairly automated, especially for block handwriting: taking ≈ 30 sec. to annotate block phrases and ≈ 1.5 min. for cursive phrases.

7 Other Related Work

There is, of course, a vast body of past work on the topic of signature verification (see [27] for a comprehensive if somewhat dated survey, [11] for a more up-to-date look at the field). However, to the best of our knowledge, there is relatively little work that encompass our goals and attack models described herein.

Perhaps the work closest to ours, although it predominately involves signatures, is that by Vielhauer and Steinmetz [33]. They use 50 features extracted from a handwriting sample to construct a biometric hash. While they performed some preliminary testing on PIN's and passphrases, the bulk of their study is on signatures, where they evaluated features based on intrapersonal deviation, interpersonal entropy with respect to their hash function, and the correlation between these two values. That work however does not report any results for meaningful attempts at forgery (i.e, other than naïve attacks).

Also germane are a series of recent papers that have started to examine the use of dynamic handwriting for the generation of cryptographic keys. Kuan, et al. present a method based on block-cipher principles to yield cryptographic keys from signatures [12]. They test their algorithm on the standard data set from the *First International Signature Verification Competition* and report EERs between 6% and 14% if the forger has access to a stolen token. The production of skilled forgeries in the SVC data set [37] resembles part of the methodol-
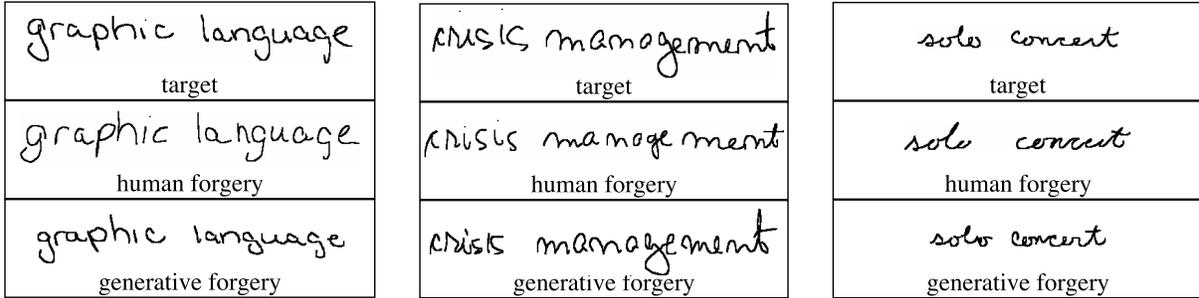
Figure 4: Example generative forgeries against block, mixed and cursive forgers. For each box, the second rendering is a trained human-generated forgery of the first, and the third was created by our generative algorithm.

ogy used in `round II` of our studies and so does not account for motivation, training, or talent.

In the realm of signature verification we also note work on an attack based on hill-climbing, but that makes the assumption that the system reveals how close of a match the input is [36]. We believe this to be clearly unrealistic, and our attack models are chosen to be more pragmatic than this.

Finally, there have been a handful of works on using generative models to attack biometric authentication. However, we note there exists significant disagreement in the literature concerning the potential effectiveness of similar (but inherently simpler) attacks on speaker verification systems (e.g., [26, 21]). Lindberg and Blomberg, for example, determined that synthesized passphrases were not effective in their small-scale experiments [15], whereas Masuko et al. found that their system was easily defeated [20].

8 Conclusions

Several fundamental computer security mechanisms rest on the ability of an intended user to generate an input that an attacker is unable to reproduce. In the biometric community, the security of biometric-based technologies hinges on this perceived inability of the attacker to reproduce the target user's input. In particular, the evaluation of biometric technologies is usually conducted under fairly weak adversarial conditions. Unfortunately, this practice may significantly underestimate the real risk of accepting forgeries as authentic. To directly address this limitation we present an automated technique for producing generative forgeries that assists in the evaluation of biometric systems. We show that our generative approach matches or exceeds the effectiveness of forgeries rendered by trained humans in our study.

Our hope is that this work will serve as a solid foundation for the work of other researchers and practitioners, particularly as it pertains to evaluating biometric au-

thentication or key-generation systems. Admittedly, such evaluations are difficult to undertake due to the reliance of recruiting large numbers of human subjects. In that regard, the generative approach presented herein should reduce the difficulty of this task and allow for more rigorous evaluations as it pertains to biometric security.

Additionally, there is much future work related to the topics presented here. For instance, although the forgeries generated by our trained forgers were alarmingly successful, it remains unclear as to the extent to which these forgeries would fool human judges, including for example, forensic document examiners. Exploring this question is one of our short term goals. Lastly, there are several directions for incorporating more sophisticated generative algorithms into our evaluation paradigm. We hope to explore these in the coming months.

Acknowledgments

Notes

[1] Although the biometric literature often refers to static or dynamic forgeries as skilled forgeries, here we make a distinction between these three types. For example, despite access to static or dynamic information, a weak forger might not be able to successfully replicate another user's writing.

[2] It is interesting to note, however, that each strong feature as defined in [33] may be inferred from our best features. However, we did find several other features that were not included in the original work.

References

[1] The biometrics consortium. http://www.biometrics.org/.

[2] Y.-J. Chang, W. Zhung, and T. Chen. Biometrics-based cryptographic key generation. In *Proceedings of the International Conference on Multimedia and Expo*, volume 3, pages 2203–2206, 2004.

[3] G. R. Doddington, W. Liggett, A. F. Martin, M. Przybocki, and D. A. Reynolds. Sheep, goats, lambs and wolves: A statistical analysis of speaker performance in the NIST 1998 speaker recognition evaluation. In *Proceedings of the Fifth International Conference on Spoken Language Processing*, November 1998.

[4] Y. Dodis, L. Reyzin, and A. Smith. Fuzzy extractors: How to generate strong keys from biometrics and other noisy data. In *Advances in Cryptology—EUROCRYPT 2004*, pages 523–540, 2004.

[5] S. J. Elliott. Development of a biometric testing protocol for dynamic signature verification. In *Proceedings of the International Conference on Automation, Robotics, and Computer Vision*, pages 782–787, Singapore, 2002.

[6] M. C. Fairhurst. Signature verification revisited: promoting practical exploitation of biometric technology. *Electronics & Communication Engineering Journal*, pages 273–280, December 1997.

[7] A. Goh and D. C. L. Ngo. Computation of cryptographic keys from face biometrics. In *Proceedings of Communications and Multimedia Security*, pages 1–13, 2003.

[8] R. M. Guest. The repeatability of signatures. In *Proceedings of the Ninth International Workshop on Frontiers in Handwriting Recognition*, pages 492–497, October 2004.

[9] I. Guyon. Handwriting synthesis from handwritten glyphs. In *Proceedings of the Fifth International Workshop on Frontiers of Handwriting Recognition*, pages 140–153, Colchester, England, 1996.

[10] C. Hertel and H. Bunke. A set of novel features for writer identification. In *Proceedings of the International Conference on Audio- and Video-based Biometric Person Authentication*, pages 679–687. Guilford, UK, 2003.

[11] A. K. Jain, F. D. Griess, and S. D. Connell. On-line signature verification. *Pattern Recognition*, 35(12):2963–2972, 2002.

[12] Y. W. Kuan, A. Goh, D. Ngo, and A. Teoh. Cryptographic keys from dynamic hand-signatures with biometric security preservation and replaceability. In *Proceedings of the Fourth IEEE Workshop on Automatic Identification Advanced Technologies*, pages 27–32, Los Alamitos, CA, 2005. IEEE Computer Society.

[13] F. Leclerc and R. Plamondon. Automatic signature verification: the state of the art 1989-1993. *International Journal of Pattern Recognition and Artificial Intelligence*, 8(3):643–660, 1994.

[14] L. Lee, T. Berger, and E. Aviczer. Reliable on-line human signature verification systems. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 18(6):643–647, June 1996.

[15] J. Lindberg and M. Blomberg. Vulnerability in speaker verification – a study of technical impostor techniques. In *Proceedings of the European Conference on Speech Communication and Technology*, volume 3, pages 1211–1214, Budapest, Hungary, September 1999.

[16] D. P. Lopresti and J. D. Raim. The effectiveness of generative attacks on an online handwriting biometric. In *Proceedings of the International Conference on Audio- and Video-based Biometric Person Authentication*, pages 1090–1099. Hilton Rye Town, NY, USA, 2005.

[17] A. J. Mansfield and J. L. Wayman. Best practices in testing and reporting performance of biometric devices. Technical Report NPL Report CMSC 14/02, Centre for Mathematics and Scientific Computing, National Physical Laboratory, August 2002.

[18] U.-V. Marti, R. Messerli, and H. Bunke. Writer identification using text line based features. In *Proceedings of the Sixth International Conference on Document Analysis and Recognition*, pages 101–105, September 2001.

[19] T. Masuko, T. Hitotsumatsu, K. Tokuda, and T. Kobayashi. On the security of hmm-based speaker verification systems against imposture using synthetic speech. In *Proceedings of the European Conference on Speech Communication and Technology*, volume 3, pages 1223–1226, Budapest, Hungary, September 1999.

[20] T. Masuko, K. Tokuda, and T. Kobayashi. Imposture using synthetic speech against speaker verification based on spectrum and pitch. In *Proceedings of the International Conference on Spoken Language Processing*, volume 3, pages 302–305, Beijing, China, October 2000.

[21] F. Monrose, M. Reiter, Q. Li, D. Lopresti, and C. Shih. Towards speech-generated cryptographic keys on resource-constrained devices. In *Proceedings of the Eleventh USENIX Security Symposium*, pages 283–296, 2002.

[22] F. Monrose, M. K. Reiter, Q. Li, and S. Wetzel. Cryptographic key generation from voice (extended abstract). In *Proceeedings of the 2001 IEEE Symposium on Security and Privacy*, pages 12–25, May 2001.

[23] F. Monrose, M. K. Reiter, and S. Wetzel. Password hardening based on keystroke dynamics. *International Journal of Information Security*, 1(2):69–83, February 2002.

[24] I. Nakanishi, H. Sakamoto, Y. Itoh, and Y. Fukui. Optimal user weighting fusion in DWT domain on-line signature verification. In *Proceedings of the International Conference on Audio- and Video-based Biometric Person Authentication*, pages 758–766. Hilton Rye Town, NY, USA, 2005.

[25] W. Nelson and E. Kishon. Use of dynamic features for signature verification. In *Proceedings of the IEEE International Conference on Systems, Man, and Cybernetics*, pages 1504–1510, October 1991.

[26] B. L. Pellom and J. H. L. Hansen. An experimental study of speaker verification sensitivity to computer voice altered imposters. In *Proceedings of the 1999 International Conference on Acoustics, Speech, and Signal Processing*, March 1999.

[27] R. Plamondon, editor. *Progress in Automatic Signature Verification*. World Scientific, 1994.

[28] R. Plamondon and G. Lorette. Automatic signature verification and writer identification – the state of the art. volume 22, pages 107–131, 1989.

[29] R. Plamondon and S. N. Srihari. On-line and off-line handwriting recognition: a comprehensive survey. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 22(1):63–84, 2000.

[30] C. Soutar, D. Roberge, A. Stoianov, R. Gilroy, and B. V. Kumar. Biometric encryption^{TM} using image processing. In *Optical Security and Counterfeit Deterrence Techniques II*, volume 3314, pages 178–188. IS&T/SPIE, 1998.

[31] U. Uludag and A. K. Jain. Fingerprint minutiae attack system. In *The Biometric Consortium Conference*, September 2004.

[32] U. Uludag, S. Pankanti, S. Prabhakar, and A. K. Jain. Biometric cryptosystems: Issues and challenges. *Proceedings of the IEEE: Special Issue on Multimedia Security of Digital Rights Management*, 92(6):948–960, 2004.

[33] C. Vielhauer and R. Steinmetz. Handwriting: Feature correlation analysis for biometric hashes. *EURASIP Journal on Applied Signal Processing*, 4:542–558, 2004.

[34] C. Vielhauer, R. Steinmetz, and A. Mayerhofer. Biometric hash based on statistical features of online signatures. In *Proceedings of the Sixteenth International Conference on Pattern Recognition*, volume 1, pages 123–126, 2002.

[35] C. Vielhauer and F. Zöbisch. A test tool to support brute-force online and offline signature forgery tests on mobile devices. In *Proceedings of the International Conference on Multimedia and Expo*, volume 3, pages 225–228, 2003.

[36] Y. Yamazaki, A. Nakashima, K. Tasaka, and N. Komatsu. A study on vulnerability in on-line writer verification system. In *Proceedings of the Eighth International Conference on Document Analysis and Recognition*, pages 640–644, Seoul, South Korea, August-September 2005.

[37] D.-Y. Yeung, H. Chang, Y. Xiong, S. George, R. Kashi, T. Matsumoto, and G. Rigoll. SVC2004: First international signature verification competition. In *Proceedings of the International Conference on Biometric Authentication (ICBA)*, Hong Kong, July 2004.

A Features

Using the quality metric, Q, as described in §4.3 we narrowed 144 state of the art features to the 36 most useful features (see Table 1). The 15 static features consisted of: the number of strokes used in rendering the phrase, the number of local horizontal and vertical extrema, and the integrated area to the left and below the writing [33]. Additional static features included the writing width and height, the total distance travelled by the pen on and off the tablet, the total area enclosed within writing loops, and the vertical centroid of these loops [8]. We also considered the distance between the upper (lower) baseline and the top (bottom) line [18], the median strokeslant [18], and the distance between the last x (y) coordinate and the maximum x (y) coordinate [14]. Note that these final two features could be considered dynamic as one may not know which coordinate is the last one rendered without access to timing information.

The 21 dynamic features consisted of: The total time spent writing, the ratio of pen-up time to pen-down time, the median pen velocity, the number of times the pen ceases to move horizontally (vertically), and the total time spent moving to the left, right, up, and down [14]. Additional dynamic features included the time of occurrence of the following events: maximum pen velocity, maximum pen velocity in the horizontal (vertical) direction, minimum velocity in the horizontal (vertical) direction, and the maximum stroke slant [14]. Finally, we considered six invariant moments of the writing, which measure the number of samples, horizontal (vertical) mass, diagonality, and horizontal (vertical) divergence [8].

Feature (f)	Description	$Q(f)$
	Spatial Features	
Pen-down distance	Total distance travelled by the pen-tip while touching the screen [8].	0.81
Median θ	Median stroke-slant, normalized to $\theta \in [0, \pi]$ [18].	0.71
Vert. end dist.	Distance between the last y-coordinate and maximum y-coordinate [14].	0.67
Y-Area	Integrated area beneath the writing [33].	0.65
Writing width	Total width of the writing [33, 8].	0.65
Writing height	Total height of the writing [33, 8].	0.65
Pen-up distance	Euclidean distance between pen-up and pen-down events.	0.64
# of strokes	Number of strokes used to render the passphrase [33].	0.63
# of extrema	Number of local extrema in the horizontal and vertical directions [33].	0.62
Lower zone	Distance between baseline and bottomline of the writing [18].	0.62
X-Area	Integrated area to the left of the writing [33].	0.62
Loop y centroid	The average value of all y coordinates contained within writing loops [8].	0.62
Loop area	Total area enclosed within loops generated by overlapping strokes [8].	0.61
Upper zone	Distance between upper-baseline and topline of the writing [18].	0.61
Horiz. end dist.	Distance between the last x-coordinate and maximum x-coordinate [14].	0.60
	Temporal Features	
Time	Total time spent writing (measured in ms) [14].	0.87
# of times $v_x = 0$	Number of times the pen ceases to move horizontally [14].	0.86
# of times $v_y = 0$	Number of times the pen ceases to move vertically [14].	0.85
Inv. Mom. 00	$\sum_x \sum_y f(x,y)$; $f(x,y) = 1$ if there is a point at (x,y) and 0 otherwise [8].	0.85
Inv. Mom. 10	$\sum_x \sum_y f(x,y) \cdot x$. Measures the horizontal mass of the writing [8].	0.82
Inv. Mom. 01	$\sum_x \sum_y f(x,y) \cdot y$. Measures the vertical mass of the writing [8].	0.79
Inv. Mom. 11	$\sum_x \sum_y f(x,y) \cdot xy$. Measures diagonality of the writing sample [8].	0.78
Time of max v_x	Time of the maximum pen-velocity in the horizontal direction [14].	0.78
Inv. Mom. 21	$\sum_x \sum_y f(x,y) \cdot x^2 y$. Measures vertical divergence [8].	0.76
Inv. Mom. 12	$\sum_x \sum_y f(x,y) \cdot xy^2$. Measures horizontal divergence [8].	0.75
Median pen velocity	Median speed of the pen-tip [14].	0.74
Duration $v_x > 0$	Total time the pen spends moving to the right [14].	0.73
Duration $v_y > 0$	Total time the pen spends moving to the up [14].	0.73
Time of max vel.	Time of the maximum pen-velocity [14].	0.72
Pen up/down ratio	Ratio time spent with the pen off and on the tablet [14].	0.71
Time of max θ	Time of maximum stroke slant.	0.70
Duration $v_y < 0$	Total time the pen spends moving to the down [14].	0.70
Duration $v_x < 0$	Total time the pen spends moving to the left [14].	0.69
Time of min v_x	Time of the minimum pen-velocity in the horizontal direction [14].	0.69
Time of min v_y	Time of the minimum pen-velocity in the vertical direction [14].	0.68
Time of max v_y	Time of the maximum pen-velocity in the vertical direction [14].	0.68

Table 1: The statistical features used to evaluate the biometric authentication system. Features were chosen based on the quality score Q defined in §4.3. θ is the angle of a given stroke, v, v_x, v_y are overall, horizontal, and vertical velocity, respectively.