1. (25 points) Prove that the language $\text{SPACETM}$ defined in the textbook (page 83, Definition (4.3)) is $\text{PSPACE}$-complete.

2. (25 points) Show that in every finite two-person game with perfect information (by finite we mean that there is an a priori upper bound $n$ on the number of moves after which the game is over and one of the two players is declared the victor—there are no draws) one of the two players has a winning strategy.

3. (25 points) Define $\text{polyL}$ to be $\bigcup_{c>0} \text{SPACE}(\log^c n)$. Steve's class $\text{SC}$ is defined to be the set of languages that can be decided by deterministic machines that run in polynomial time and $\log^c n$ space for some $c > 0$.

   It is an open problem whether $\text{PATH} \in \text{SC}$. Why does Savitch's Theorem not resolve this question? Is $\text{SC}$ the same as $\text{polyL} \cap \text{P}$?

4. (25 points) Let $\varphi$ be a 2CNF formula with exactly two literals per clause. Let $x_1, \ldots, x_n$ be the variables in $\varphi$. Associate with $\varphi$ a directed graph $G_\varphi = (V, E)$, where

   $$V = \{ x_1, \overline{x_1}, x_2, \overline{x_2}, \ldots, x_n, \overline{x_n} \}$$

   (i.e., $V$ is the set of all literals that may appear in $\varphi$), and a pair $(t_1, t_2)$ is an edge in $G_\varphi$ iff $(\overline{t_1} \lor t_2)$ is a clause in $\varphi$.

   (a) Show that $\varphi$ is unsatisfiable iff there is a directed cycle in $G_\varphi$ in which both $x_i$ and $\overline{x_i}$ appear, for some variable $x_i$.

   (b) Use part a) to show that $\text{2SAT}$ is in $\text{NL}$.

   (c) Show that $\overline{\text{PATH}} \leq_\ell \text{2SAT}$. Use this and part b) to show that $\text{2SAT}$ is $\text{NL}$-complete.