1. (20 points) Let \( \text{HALT} \) be the Halting language defined in Theorem 1.11 in the textbook. Show that \( \text{HALT} \) is \( \text{NP} \)-hard. Is it \( \text{NP} \)-complete?

2. (20 points) Suppose \( L_1, L_2 \in \text{NP} \). Then is \( L_1 \cup L_2 \in \text{NP} \)? What about \( L_1 \cap L_2 \)?

3. (20 points) Show that, if \( \mathbf{P} = \mathbf{NP} \), then every language \( A \in \mathbf{P} \), except \( A = \emptyset \) and \( A = \Sigma^* \), is \( \text{NP} \)-complete. Here \( \Sigma \) is the alphabet, and you may assume that it is \( \{0, 1\} \).

4. (20 points) A subset of the nodes of a graph \( G \) is a dominating set if every other node of \( G \) is adjacent to some node in the subset. Let

\[
\text{DOMINATING-SET} = \{ \langle G, k \rangle | \text{G has a dominating set with } k \text{ nodes} \}.
\]

Show that it is \( \text{NP} \)-complete by giving a reduction from \( \text{VERTEX-COVER} \).

5. (20 points) Prove that if \( \mathbf{P} = \mathbf{NP} \), then \( \mathbf{NP} = \text{coNP} \).